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Self-interactions in a topological BF-type model in $D = 5$

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Abstract: All consistent interactions in five spacetime dimensions that can be added to a free BF-type model involving one scalar field, two types of one-forms, two sorts of two-forms, and one three-form are investigated by means of deforming the solution to the master equation with the help of specific cohomological techniques. The couplings are obtained on the grounds of smoothness, locality, (background) Lorentz invariance, Poincaré invariance, and the preservation of the number of derivatives on each field.

Keywords: BRST Quantization, BRST Symmetry.

Contents

1. Introduction

The power of the BRST formalism was strongly increased by its cohomological development, which allowed, among others, a useful investigation of many interesting aspects related to the perturbative renormalization problem [1 – 5], anomaly-tracking mechanism [5 – 10], simultaneous study of local and rigid invariances of a given theory [11], as well as to the reformulation of the construction of consistent interactions in gauge theories [12 – 16] in terms of the deformation theory [17 – 21], or, actually, in terms of the deformation of the solution to the master equation.

The main aim of this paper is to construct the consistent interactions in five spacetime dimensions that can be added to a free BF-type model [22] involving one scalar field, two types of one-forms, two sorts of two-forms, and one three-form by means of deforming the solution to the master equation with the help of specific cohomological techniques. Interacting topological field theories of BF-type are important in view of their relationship with Poisson Sigma Models, which are known to explain interesting aspects of two-dimensional gravity, including the study of classical solutions [23 – 31]. Various aspects of BF models can be found in [32 – 36]. The present paper extends our former hamiltonian results [37, 38] on BF-type models.

The couplings are obtained on the grounds of smoothness, locality, (background) Lorentz invariance, Poincaré invariance, and the preservation of the number of derivatives on each field. The starting, free BF model possesses abelian gauge transformations, which are off-shell, third-order reducible. The entire lagrangian formulation of the interacting theory is obtained from the computation of the deformed solution to the master equation, order by order in the coupling constant g. Thus, the first-order deformation of the solution to the master equation is parametrized by seven arbitrary, smooth functions of the undifferentiated scalar field. The consistency of the deformation procedure at order g^2 imposes some restrictions on the above mentioned functions, which lead to three kinds of interacting models that are in a way complementary to each other. In all situations the fully deformed solution to the master equation that is consistent to all orders in the coupling constant stops at order one in g. Related to the three types of interacting BF theories, all of them describe a deformed model with an open gauge algebra, which closes on-shell (on the stationary surface of deformed field equations). At the level of reducibility relations, the first coupled model possesses on-shell first- and second-order reducibility relations, the second interacting theory exhibits on-shell reducibility relations to all (the three) orders, while in the last situation only the first-order reducibility relations close on-shell.

The paper is organized into six sections. Section 2 introduces the model to be considered and constructs its free lagrangian BRST symmetry. Section 3 briefly reviews the procedure of adding consistent interactions in gauge theories based on the deformation of the solution to the master equation. In section 4 we construct the lagrangian interactions for the starting free system in five dimensions by solving the deformation equations with the help of standard cohomological techniques. Section 5 discusses the resulting interacting models and section 6 ends the paper with the main conclusions. The paper also contains seven appendix sections including various aspects mentioned in the main text.

2. Free BRST differential

We start from a free, five-dimensional BF-like theory involving one scalar field φ, two types of one-forms H_μ and A_μ, two sorts of two-forms $B_{\mu\nu}$ and $\phi_{\mu\nu}$, and one three-form $K_{\mu\nu\rho}$

$$S_0^{\mathrm{L}}\left[\varphi, H, A, B, \phi, K\right] = \int d^5x \left(H_\mu \partial^\mu \varphi + \frac{1}{2} B^{\mu\nu} \partial_{[\mu} A_{\nu]} + \frac{1}{3} K^{\mu\nu\rho} \partial_{[\mu} \phi_{\nu\rho]} \right). \tag{2.1}$$

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Action (2.1) is found invariant under the gauge transformations

$$\delta_{\epsilon,\xi}A^{\mu} = \partial^{\mu}\epsilon\,, \qquad \delta_{\epsilon,\xi}H^{\mu} = 2\partial_{\nu}\epsilon^{\mu\nu}\,, \qquad \delta_{\epsilon,\xi}\varphi = 0\,, \tag{2.2}$$

$$\delta_{\epsilon,\xi}B^{\mu\nu} = -3\partial_{\rho}\epsilon^{\mu\nu\rho}\,, \qquad \delta_{\epsilon,\xi}\phi_{\mu\nu} = \partial_{[\mu}\xi_{\nu]}\,, \qquad \delta_{\epsilon,\xi}K^{\mu\nu\rho} = 4\partial_{\lambda}\xi^{\mu\nu\rho\lambda}\,, \tag{2.3}$$

where the gauge parameters ϵ, $\epsilon^{\mu\nu}$, $\epsilon^{\mu\nu\rho}$, ξ_{μ}, and $\xi^{\mu\nu\rho\lambda}$ are bosonic, with $\epsilon^{\mu\nu}$, $\epsilon^{\mu\nu\rho}$, and $\xi^{\mu\nu\rho\lambda}$ completely antisymmetric. By means of (2.2)–(2.3) we read the nonvanishing gauge generators (written in De Witt condensed notations)

$$(Z^{\mu}_{(A)}) = \partial^{\mu}\,, \qquad (Z^{\mu}_{(H)})_{\alpha\beta} = -\partial_{[\alpha}\,\delta^{\mu}_{\beta]}\,, \qquad (Z^{\mu\nu}_{(B)})_{\alpha\beta\gamma} = -\frac{1}{2}\partial_{[\alpha}\,\delta^{\mu}_{\beta}\delta^{\nu}_{\gamma]}\,, \tag{2.4}$$

$$(Z^{\mu\nu}_{(\phi)})_{\alpha} = \partial^{[\mu}\delta^{\nu]}_{\alpha}\,, \qquad (Z^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta} = -\frac{1}{6}\partial_{[\alpha}\,\delta^{\mu}_{\beta}\delta^{\nu}_{\gamma}\delta^{\rho}_{\delta]}\,, \tag{2.5}$$

where we put an extra lower index (A), (H), etc., in order to indicate with what field is a certain gauge generator associated. Everywhere in this paper we understand that the notation $[\alpha\beta\ldots\gamma]$ signifies complete antisymmetry with respect to the Lorentz indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. The above gauge transformations are abelian and off-shell, third-order reducible. More precisely, the gauge generators of the one-form H^{μ} are third-order reducible, with the first-, second-, and respectively third-order reducibility functions

$$(Z_1^{\alpha\beta})_{\mu'\nu'\rho'} = -\frac{1}{2}\partial_{[\mu'}\,\delta^{\alpha}_{\nu'}\delta^{\beta}_{\rho']}\,, \qquad (Z_2^{\mu'\nu'\rho'})_{\alpha'\beta'\gamma'\delta'} = -\frac{1}{6}\partial_{[\alpha'}\,\delta^{\mu'}_{\beta'}\delta^{\nu'}_{\gamma'}\delta^{\rho'}_{\delta']}\,, \tag{2.6}$$

$$(Z_3^{\alpha'\beta'\gamma'\delta'})_{\mu''\nu''\rho''\lambda''\sigma''} = -\frac{1}{24}\partial_{[\mu''}\,\delta^{\alpha'}_{\nu''}\delta^{\beta'}_{\rho''}\delta^{\gamma'}_{\lambda''}\delta^{\delta'}_{\sigma'']}, \tag{2.7}$$

the gauge generators of the two-form $B^{\mu\nu}$ are second-order reducible, with the reducibility functions

$$(Z_1^{\alpha\beta\gamma})_{\mu'\nu'\rho'\lambda'} = -\frac{1}{6}\partial_{[\mu'}\,\delta^{\alpha}_{\nu'}\delta^{\beta}_{\rho'}\delta^{\gamma}_{\lambda']}\,, \qquad (Z_2^{\mu'\nu'\rho'\lambda'})_{\alpha\beta\gamma\delta\varepsilon} = -\frac{1}{24}\partial_{[\alpha}\,\delta^{\mu'}_{\beta}\,\delta^{\nu'}_{\gamma}\,\delta^{\rho'}_{\delta}\,\delta^{\lambda'}_{\varepsilon]}\,, \tag{2.8}$$

while the gauge generators of the two-form $\phi_{\mu\nu}$ and of the three-form $K^{\mu\nu\rho}$ are first-order reducible, with the corresponding reducibility functions respectively of the form

$$(Z_1^{\alpha}) = \partial^{\alpha}\,, \qquad (Z_1^{\alpha\beta\gamma\delta})_{\mu'\nu'\rho'\lambda'\sigma'} = -\frac{1}{24}\partial_{[\mu'}\,\delta^{\alpha}_{\nu'}\delta^{\beta}_{\rho'}\delta^{\gamma}_{\lambda'}\delta^{\delta}_{\sigma']}\,. \tag{2.9}$$

The concrete form of the reducibility relations written in condensed De Witt notations are expressed as follows. The first-order reducibility relations are

$$(Z^{\mu}_{(H)})_{\alpha\beta}(Z_1^{\alpha\beta})_{\mu'\nu'\rho'} = 0, \qquad (Z^{\mu\nu}_{(B)})_{\alpha\beta\gamma}(Z_1^{\alpha\beta\gamma})_{\mu'\nu'\rho'\lambda'} = 0\,, \tag{2.10}$$

$$(Z^{\mu\nu}_{(\phi)})_{\alpha}(Z_1^{\alpha}) = 0\,, \qquad (Z^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta}(Z_1^{\alpha\beta\gamma\delta})_{\mu'\nu'\rho'\lambda'\sigma'} = 0\,, \tag{2.11}$$

the second-order ones read as

$$(Z_1^{\alpha\beta})_{\mu'\nu'\rho'}(Z_2^{\mu'\nu'\rho'})_{\alpha'\beta'\gamma'\delta'} = 0\,, \qquad (Z_1^{\alpha\beta\gamma})_{\mu'\nu'\rho'\lambda'}(Z_2^{\mu'\nu'\rho'\lambda'})_{\alpha\beta\gamma\delta\varepsilon} = 0\,, \tag{2.12}$$

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while the third-order reducibility relations can be written as

$$\left(Z_2^{\mu'\nu'\rho'}\right)_{\alpha'\beta'\gamma'\delta'}\left(Z_3^{\alpha'\beta'\gamma'\delta'}\right)_{\mu''\nu''\rho''\lambda''\sigma''}=0\,. \tag{2.13}$$

We observe that the BF-like theory under study is a usual linear gauge theory (its field equations are linear in the fields and first-order in their spacetime derivatives), whose generating set of gauge transformations is third-order reducible, such that we can define in a consistent manner its Cauchy order, which is found to be equal to five.

In order to construct the BRST symmetry of this free theory, we introduce the field/ghost and antifield spectra

$$\Phi^{\alpha_0}=\left(A^\mu,H^\mu,\varphi,B^{\mu\nu},K^{\mu\nu\rho},\phi_{\mu\nu}\right),\qquad \Phi^*_{\alpha_0}=\left(A^*_\mu,H^*_\mu,\varphi^*,B^*_{\mu\nu},K^*_{\mu\nu\rho},\phi^{*\mu\nu}\right), \tag{2.14}$$

$$\eta^{\alpha_1}=\left(\eta,C^{\mu\nu},\eta^{\mu\nu\rho},\mathcal{G}^{\mu\nu\rho\lambda},C_\mu\right),\qquad \eta^*_{\alpha_1}=\left(\eta^*,C^*_{\mu\nu},\eta^*_{\mu\nu\rho},\mathcal{G}^*_{\mu\nu\rho\lambda},C^{*\mu}\right), \tag{2.15}$$

$$\eta^{\alpha_2}=\left(C^{\mu\nu\rho},\eta^{\mu\nu\rho\lambda},\mathcal{G}^{\mu\nu\rho\lambda\sigma},C\right),\qquad \eta^*_{\alpha_2}=\left(C^*_{\mu\nu\rho},\eta^*_{\mu\nu\rho\lambda},\mathcal{G}^*_{\mu\nu\rho\lambda\sigma},C^*\right), \tag{2.16}$$

$$\eta^{\alpha_3}=\left(C^{\mu\nu\rho\lambda},\eta^{\mu\nu\rho\lambda\sigma}\right),\qquad \eta^*_{\alpha_3}=\left(C^*_{\mu\nu\rho\lambda},\eta^*_{\mu\nu\rho\lambda\sigma}\right), \tag{2.17}$$

$$\eta^{\alpha_4}=\left(C^{\mu\nu\rho\lambda\sigma}\right),\qquad \eta^*_{\alpha_4}=\left(C^*_{\mu\nu\rho\lambda\sigma}\right). \tag{2.18}$$

The fermionic ghosts η^{α_1} respectively correspond to the bosonic gauge parameters $\epsilon^{\alpha_1}=\left(\epsilon,\epsilon^{\mu\nu},\epsilon^{\mu\nu\rho},\xi^{\mu\nu\rho\lambda},\xi_\mu\right)$, the bosonic ghosts for ghosts η^{α_2} are due to the first-order reducibility relations (2.10)–(2.11), the fermionic ghosts for ghosts for ghosts η^{α_3} are required by the second-order reducibility relations (2.12), while the bosonic ghosts for ghosts for ghosts for ghosts η^{α_4} are imposed by the third-order reducibility relations (2.13). The star variables represent the antifields of the corresponding fields/ghosts. Their Grassmann parities are obtained via the usual rule $\varepsilon\left(\chi^*_\Delta\right)=\left(\varepsilon\left(\chi^\Delta\right)+1\right)\mod 2$, where we employed the notations

$$\chi^\Delta=\left(\Phi^{\alpha_0},\eta^{\alpha_1},\eta^{\alpha_2},\eta^{\alpha_3},\eta^{\alpha_4}\right),\qquad \chi^*_\Delta=\left(\Phi^*_{\alpha_0},\eta^*_{\alpha_1},\eta^*_{\alpha_2},\eta^*_{\alpha_3},\eta^*_{\alpha_4}\right). \tag{2.19}$$

Since both the gauge generators and the reducibility functions are field-independent, it follows that the BRST differential reduces to $s=\delta+\gamma$, where δ is the Koszul-Tate differential, and γ means the exterior longitudinal derivative. The Koszul-Tate differential is graded in terms of the antighost number [agh, $\mathrm{agh}(\delta)=-1$, $\mathrm{agh}(\gamma)=0$] and enforces a resolution of the algebra of smooth functions defined on the stationary surface of field equations for the action (2.1), $C^\infty(\Sigma)$, $\Sigma:\delta S_0^{\mathrm{L}}/\delta\Phi^{\alpha_0}=0$. The exterior longitudinal derivative is graded in terms of the pure ghost number [pgh, $\mathrm{pgh}(\gamma)=1$, $\mathrm{pgh}(\delta)=0$] and is correlated with the original gauge symmetry via its cohomology in pure ghost number zero computed in $C^\infty(\Sigma)$, which is isomorphic to the algebra of physical observables for this free theory. These two degrees of the generators (2.14)–(2.18) from the BRST complex are valued like

$$\mathrm{pgh}\left(\Phi^{\alpha_0}\right)=0,\qquad \mathrm{pgh}\left(\eta^{\alpha_k}\right)=k,\qquad \mathrm{pgh}\left(\Phi^*_{\alpha_0}\right)=0,\qquad \mathrm{pgh}\left(\eta^*_{\alpha_k}\right)=0, \tag{2.20}$$

$$\mathrm{agh}\left(\Phi^{\alpha_0}\right)=0,\qquad \mathrm{agh}\left(\eta^{\alpha_k}\right)=0,\qquad \mathrm{agh}\left(\Phi^*_{\alpha_0}\right)=1,\qquad \mathrm{agh}\left(\eta^*_{\alpha_k}\right)=k+1, \tag{2.21}$$

for $k=\overline{1,4}$. The actions of the differentials δ and γ on the above generators read as

$$\delta\Phi^{\alpha_0} = 0\,, \quad \delta\eta^{\alpha_k} = 0 \quad (k=\overline{1,4})\,, \tag{2.22}$$

$$\delta A^*_\mu = -\partial^\nu B_{\mu\nu}\,, \quad \delta H^*_\mu = -\partial_\mu\varphi\,, \quad \delta\varphi^* = \partial^\mu H_\mu\,, \quad \delta B^*_{\mu\nu} = -\frac{1}{2}\partial_{[\mu}A_{\nu]}\,, \tag{2.23}$$

$$\delta\phi^*_{\mu\nu} = \partial^\rho K_{\mu\nu\rho}\,, \quad \delta K^*_{\mu\nu\rho} = -\frac{1}{3}\partial_{[\mu}\phi_{\nu\rho]}\,, \tag{2.24}$$

$$\delta\eta^* = -\partial^\mu A^*_\mu\,, \quad \delta C^*_{\mu\nu} = \partial_{[\mu}H^*_{\nu]}\,, \quad \delta\eta^*_{\mu\nu\rho} = \partial_{[\mu}B^*_{\nu\rho]}\,, \tag{2.25}$$

$$\delta C^*_\mu = 2\partial^\nu\phi^*_{\mu\nu}\,, \quad \delta\mathcal{G}^*_{\mu\nu\rho\lambda} = \partial_{[\mu}K^*_{\nu\rho\lambda]}\,, \quad \delta C^*_{\mu\nu\rho} = -\partial_{[\mu}C^*_{\nu\rho]}\,, \tag{2.26}$$

$$\delta\eta^*_{\mu\nu\rho\lambda} = -\partial_{[\mu}\eta^*_{\nu\rho\lambda]}\,, \quad \delta C^* = \partial^\mu C^*_\mu\,, \quad \delta\mathcal{G}^*_{\mu\nu\rho\lambda\sigma} = -\partial_{[\mu}\mathcal{G}^*_{\nu\rho\lambda\sigma]}\,, \tag{2.27}$$

$$\delta C^*_{\mu\nu\rho\lambda} = \partial_{[\mu}C^*_{\nu\rho\lambda]}\,, \quad \delta\eta^*_{\mu\nu\rho\lambda\sigma} = \partial_{[\mu}\eta^*_{\nu\rho\lambda\sigma]}\,, \quad \delta C^*_{\mu\nu\rho\lambda\sigma} = -\partial_{[\mu}C^*_{\nu\rho\lambda\sigma]}\,, \tag{2.28}$$

$$\gamma\Phi^*_{\alpha_0} = 0\,, \quad \gamma\eta^*_{\alpha_k} = 0 \quad (k=\overline{1,4})\,, \tag{2.29}$$

$$\gamma A^\mu = \partial^\mu\eta\,, \quad \gamma H^\mu = 2\partial_\nu C^{\mu\nu}\,, \quad \gamma B^{\mu\nu} = -3\partial_\rho\eta^{\mu\nu\rho}\,, \quad \gamma\varphi = 0\,, \tag{2.30}$$

$$\gamma\phi_{\mu\nu} = \partial_{[\mu}C_{\nu]}\,, \quad \gamma K^{\mu\nu\rho} = 4\partial_\lambda\mathcal{G}^{\mu\nu\rho\lambda}\,, \quad \gamma\eta = 0\,, \quad \gamma C^{\mu\nu} = -3\partial_\rho C^{\mu\nu\rho}\,, \tag{2.31}$$

$$\gamma\eta^{\mu\nu\rho} = 4\partial_\lambda\eta^{\mu\nu\rho\lambda}\,, \quad \gamma C_\mu = \partial_\mu C\,, \quad \gamma\mathcal{G}^{\mu\nu\rho\lambda} = -5\partial_\sigma\mathcal{G}^{\mu\nu\rho\lambda\sigma}\,, \tag{2.32}$$

$$\gamma C^{\mu\nu\rho} = 4\partial_\lambda C^{\mu\nu\rho\lambda}\,, \quad \gamma\eta^{\mu\nu\rho\lambda} = -5\partial_\sigma\eta^{\mu\nu\rho\lambda\sigma}\,, \quad \gamma C = \gamma\mathcal{G}^{\mu\nu\rho\lambda\sigma} = 0\,, \tag{2.33}$$

$$\gamma C^{\mu\nu\rho\lambda} = -5\partial_\sigma C^{\mu\nu\rho\lambda\sigma}\,, \quad \gamma\eta^{\mu\nu\rho\lambda\sigma} = 0\,, \quad \gamma C^{\mu\nu\rho\lambda\sigma} = 0\,. \tag{2.34}$$

The overall degree that grades the BRST complex is named ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = \mathrm{gh}(s) = 1$.

The BRST symmetry admits a canonical action $s\cdot = \left(\cdot, \bar{S}\right)$, where its canonical generator $\left[\mathrm{gh}\left(\bar{S}\right) = 0,\ \varepsilon\left(\bar{S}\right) = 0\right]$ satisfies the classical master equation $\left(\bar{S}, \bar{S}\right) = 0$. The symbol $(,)$ denotes the antibracket, defined by decreeing the fields/ghosts conjugated with the corresponding antifields. In the case of the free theory under discussion, the solution to the master equation takes the form

$$\begin{aligned}\bar{S} = S_0^{\mathrm{L}} + \int d^5x \Big[& A^*_\mu\partial^\mu\eta + 2H^*_\mu\partial_\nu C^{\mu\nu} - 3B^*_{\mu\nu}\partial_\rho\eta^{\mu\nu\rho} + \phi^{*\mu\nu}\partial_{[\mu}C_{\nu]} + \\ & + 4K^*_{\mu\nu\rho}\partial_\lambda\mathcal{G}^{\mu\nu\rho\lambda} - 3C^*_{\mu\nu}\partial_\rho C^{\mu\nu\rho} + 4\eta^*_{\mu\nu\rho}\partial_\lambda\eta^{\mu\nu\rho\lambda} - \\ & - 5\mathcal{G}^*_{\mu\nu\rho\lambda}\partial_\sigma\mathcal{G}^{\mu\nu\rho\lambda\sigma} + C^{*\mu}\partial_\mu C + 4C^*_{\mu\nu\rho}\partial_\lambda C^{\mu\nu\rho\lambda} - \\ & - 5\left(\eta^*_{\mu\nu\rho\lambda}\partial_\sigma\eta^{\mu\nu\rho\lambda\sigma} + C^*_{\mu\nu\rho\lambda}\partial_\sigma C^{\mu\nu\rho\lambda\sigma}\right)\Big]\,. \end{aligned} \tag{2.35}$$

The solution to the master equation encodes all the information on the gauge structure of a given theory. We remark that in our case the solution (2.35) to the master equation breaks into terms with the antighost number ranging from zero to four. Let us briefly recall the significance of the various terms present in the solution to the master equation. Thus, the part with the antighost number equal to zero is nothing but the lagrangian action of the gauge model under study. The components of antighost number equal to one are always proportional with the gauge generators [in this situation (2.4)–(2.5)]. If the gauge algebra were non-abelian, then there would appear terms linear in the antighost number two antifields and quadratic in the pure ghost number one ghosts. The absence of such terms in our case shows that the gauge transformations are abelian. The terms from (2.35) with higher antighost number give us information on the reducibility functions (2.6)–(2.9). If the reducibility relations held on-shell, then there would appear components linear in the ghosts for ghosts (ghosts of pure ghost number strictly greater than one) and quadratic in the various antifields. Such pieces are not present in (2.35), since the reducibility relations (2.10)–(2.13) hold off-shell. Other possible components in the solution to the master equation offer information on the higher-order structure functions related to the tensor gauge structure of the theory. There are no such terms in (2.35), as a consequence of the fact that all higher-order structure functions vanish for the theory under study.

3. Deformation of the master equation: a brief review

We begin with a "free" gauge theory, described by a lagrangian action $S_0\left[\Phi^{\alpha_0}\right]$, invariant under some gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\ \ \alpha_1} \epsilon^{\alpha_1}\,, \qquad \frac{\delta S_0}{\delta \Phi^{\alpha_0}} Z^{\alpha_0}_{\ \ \alpha_1} = 0\,, \tag{3.1}$$

and consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve the field spectrum and the original number of gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the master equation corresponding to the "free" theory [17, 18]. Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If a consistent interacting gauge theory can be constructed, then the solution $\bar{S}$ to the master equation associated with the "free" theory, $\left(\bar{S}, \bar{S}\right) = 0$, can be deformed into a solution S,

$$\bar{S} \rightarrow S = \bar{S} + gS_1 + g^2 S_2 + \cdots = \bar{S} + g \int d^D x\, a + g^2 \int d^D x\, b + \cdots\,, \tag{3.2}$$

of the master equation for the deformed theory

$$(S, S) = 0\,, \tag{3.3}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The equation (3.3) splits, according to the various orders in the coupling constant (or deformation parameter) g, into

$$\left(\bar{S}, \bar{S}\right) = 0\,, \tag{3.4}$$

$$2\left(S_1, \bar{S}\right) = 0\,, \tag{3.5}$$

$$2\left(S_2, \bar{S}\right) + \left(S_1, S_1\right) = 0\,, \tag{3.6}$$

$$\left(S_3, \bar{S}\right) + \left(S_1, S_2\right) = 0\,, \tag{3.7}$$

$$\vdots$$

The equation (3.4) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, S_1, is a cocycle of the "free" BRST differential $s\cdot = \left(\cdot, \bar{S}\right)$. However, solely cohomologically nontrivial solutions to (3.5) should be taken into account, as the BRST-exact ones can be eliminated by some (in general nonlinear) field redefinitions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the "free" theory. It has been shown in [17, 18] (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations [(3.6)–(3.7), etc.]. However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques. As it will be seen below, all the interactions in the case of the model under study turn out to be local.

4. Determination of consistent interactions

In this section we determine all consistent interactions that can be added to the free theory that describes a topological BF-type model in five spacetime dimensions. This is done by means of solving the deformation equations (3.5)–(3.7), etc., by means of specific cohomological techniques in the presence of certain hypotheses to be discussed below. The interacting theory and its gauge structure are deduced from the analysis of the deformed solution to the master equation that is consistent to all orders in the deformation parameter.

4.1 Standard material: $H(\gamma)$ and $H(\delta|d)$

For obvious reasons, we consider only smooth, local, (background) Lorentz invariant, Poincaré invariant quantities (i.e., we do not allow explicit dependence on the spacetime coordinates), and, moreover, require the preservation of the number of derivatives on each field with respect to the free theory. The smoothness of the deformations refers to the fact that the deformed solution to the master equation (3.2) is smooth in the coupling constant g and reduces to the original solution (2.35) in the free limit ($g = 0$). If we make the notation $S_1 = \int d^5x\, a$, with a a local function, then the equation (3.5), which we have seen that controls the first-order deformation, takes the local form

$$sa = \partial_\mu m^\mu\,, \qquad \mathrm{gh}\,(a) = 0\,, \qquad \varepsilon\,(a) = 0\,, \tag{4.1}$$

for some local m^μ, and it shows that the non-integrated density of the first-order deformation pertains to the local cohomology of s in ghost number zero, $a \in H^0(s|d)$, where d denotes the exterior spacetime differential. The solution to the equation (4.1) is unique up to s-exact pieces plus divergences

$$a \to a + sb + \partial_\mu n^\mu\,, \qquad \mathrm{gh}\,(b) = -1\,, \qquad \varepsilon\,(b) = 1\,, \qquad \mathrm{gh}\,(n^\mu) = 0\,, \qquad \varepsilon\,(n^\mu) = 0\,. \tag{4.2}$$

At the same time, if the general solution of (4.1) is found to be completely trivial, $a = sb + \partial_\mu n^\mu$, then it can be made to vanish $a = 0$.

In order to analyze the equation (4.1), we develop a according to the antighost number

$$a = \sum_{i=1}^{I} a_i\,, \qquad \mathrm{agh}\,(a_i) = i\,, \qquad \mathrm{gh}\,(a_i) = 0\,, \qquad \varepsilon\,(a_i) = 0\,, \tag{4.3}$$

and assume, without loss of generality, that the decomposition (4.3) stops at some finite value of I. This can be shown, for instance, like in [39] (section 3), under the sole assumption that the interacting lagrangian at the first order in the coupling constant, a_0, has a finite, but otherwise arbitrary derivative order. Inserting the decomposition (4.3) into the equation (4.1) and projecting it on the various values of the antighost number, we obtain the tower of equations

$$\gamma a_I = \partial_\mu \overset{(I)^\mu}{m}\,, \tag{4.4}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)^\mu}{m}\,, \tag{4.5}$$

$$\delta a_i + \gamma a_{i-1} = \partial_\mu \overset{(i-1)^\mu}{m}\,, \qquad 1 \leq i \leq I-1\,, \tag{4.6}$$

where $\left(\overset{(i)^\mu}{m}\right)_{i=\overline{0,I}}$ are some local currents with $\mathrm{agh}\left(\overset{(i)^\mu}{m}\right) = i$. The equation (4.4) can be replaced in strictly positive values of the antighost number by

$$\gamma a_I = 0\,, \qquad I > 0\,. \tag{4.7}$$

The proof of this statement is done in Corollary 1 from the appendix A. Due to the second-order nilpotency of γ ($\gamma^2 = 0$), the solution to the equation (4.7) is clearly unique up to γ-exact contributions

$$a_I \rightarrow a_I + \gamma b_I\,, \qquad \mathrm{agh}\,(b_I) = I\,, \qquad \mathrm{pgh}\,(b_I) = I-1\,, \qquad \varepsilon\,(b_I) = 1\,. \tag{4.8}$$

Meanwhile, if it turns out that a_I exclusively reduces to γ -exact terms, $a_I = \gamma b_I$, then it can be made to vanish, $a_I = 0$. In other words, the nontriviality of the first-order deformation a is translated at its highest antighost number component into the requirement that $a_I \in H^I(\gamma)$, where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal derivative γ in pure ghost number equal to I. So, in order to solve the equation (4.1) [equivalent with (4.7) and (4.5)–(4.6)], we need to compute the cohomology of γ, $H(\gamma)$, and, as it will be made clear below, also the local homology of δ, $H(\delta|d)$.

On behalf of the definitions (2.29)–(2.34) it is simple to see that $H(\gamma)$ is spanned by

$$F_{\bar{A}} = \left(\varphi, \partial^{[\mu}A^{\nu]}, \partial_\mu H^\mu, \partial_\mu B^{\mu\nu}, \partial_{[\mu}\phi_{\nu\rho]}, \partial_\rho K^{\mu\nu\rho}\right), \tag{4.9}$$

by the antifields χ^*_Δ from (2.19), by all of their spacetime derivatives, as well as by the undifferentiated ghosts

$$\eta^{\bar{\Upsilon}} = \left(\eta, C, \mathcal{G}^{\mu\nu\rho\lambda\sigma}, \eta^{\mu\nu\rho\lambda\sigma}, C_{\mu\nu\rho\lambda\sigma}\right). \tag{4.10}$$

[The derivatives of the ghosts $\eta^{\bar{\Upsilon}}$ are removed from $H(\gamma)$ since they are γ-exact, in agreement with the first relation in (2.30), the second formula in (2.32), the last equation in (2.32), the second relation in (2.33), and the first definition from (2.34).] If we denote by $e^M\left(\eta^{\bar{\Upsilon}}\right)$ the elements with pure ghost number M of a basis in the space of the polynomials in the ghosts (4.10), it follows that the general solution to the equation (4.7) takes the form

$$a_I = \alpha_I\left(\left[F_{\bar{A}}\right], \left[\chi^*_{\Delta}\right]\right) e^I\left(\eta^{\bar{\Upsilon}}\right), \tag{4.11}$$

where $\mathrm{agh}\left(\alpha_I\right) = I$ and $\mathrm{pgh}\left(e^I\right) = I$. The notation $f([q])$ means that f depends on q and its spacetime derivatives up to a finite order. The objects α_I [obviously nontrivial in $H^0(\gamma)$] will be called "invariant polynomials". The result that we can replace the equation (4.4) with the less obvious one (4.7) is a nice consequence of the fact that the cohomology of the exterior spacetime differential is trivial in the space of invariant polynomials in strictly positive antighost numbers. For more details on invariant polynomials, see the appendix A.

Inserting (4.11) in (4.5) we obtain that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions a_{I-1} is that the invariant polynomials α_I are (nontrivial) objects from the local cohomology of Koszul-Tate differential $H(\delta|d)$ in antighost number $I>0$ and in pure ghost number zero,

$$\delta\alpha_I = \partial_\mu \overset{(I-1)^{\mu}}{j}, \qquad \mathrm{agh}\left(\overset{(I-1)^{\mu}}{j}\right) = I-1, \qquad \mathrm{pgh}\left(\overset{(I-1)^{\mu}}{j}\right) = 0. \tag{4.12}$$

We recall that the local cohomology $H(\delta|d)$ is completely trivial in both strictly positive antighost *and* pure ghost numbers (for instance, see [40], Theorem 5.4, and [41]). Using the fact that the BF model under study is a linear gauge theory of Cauchy order equal to five and the general result from [40, 41], according to which the local cohomology of the Koszul-Tate differential at pure ghost number zero is trivial in antighost numbers strictly greater than its Cauchy order, we can state that

$$H_J(\delta|d) = 0 \quad \text{for all } J>5, \tag{4.13}$$

where $H_J(\delta|d)$ represents the local cohomology of the Koszul-Tate differential in antighost number J and in zero pure ghost number. Moreover, if the invariant polynomial α_J, with $\mathrm{agh}\left(\alpha_J\right) = J \geq 5$, is trivial in $H_J(\delta|d)$, then it can be taken to be trivial also in $H_J^{\mathrm{inv}}(\delta|d)$

$$\left(\alpha_J = \delta b_{J+1} + \partial_\mu \overset{(J)^{\mu}}{c}, \ \mathrm{agh}\left(\alpha_J\right) = J \geq 5\right) \Rightarrow \alpha_J = \delta\beta_{J+1} + \partial_\mu \overset{(J)^{\mu}}{\gamma}, \tag{4.14}$$

with both β_{J+1} and $\overset{(J)^{\mu}}{\gamma}$ invariant polynomials. Here, $H_J^{\mathrm{inv}}(\delta|d)$ denotes the invariant characteristic cohomology in antighost number J (the local cohomology of the Koszul-Tate differential in the space of invariant polynomials). [An element of $H_I^{\mathrm{inv}}(\delta|d)$ is defined via an equation of the type (4.12), but with α_I and the corresponding current invariant polynomials.] The result (4.14) is proved in detail in Theorem 3 from the appendix B. It is important since, together with (4.13), ensures that the entire invariant characteristic cohomology in antighost numbers strictly greater than five is trivial

$$H_J^{\mathrm{inv}}(\delta|d) = 0 \qquad \text{for all } J>5. \tag{4.15}$$

The nontrivial representatives of $H_J(\delta|d)$ at pure ghost number zero and of $H_J^{\mathrm{inv}}(\delta|d)$ for $J\geq 2$ depend neither on $\left(\partial^{[\mu}A^{\nu]},\partial_\mu H^\mu,\partial_\mu B^{\mu\nu},\partial_{[\mu}\phi_{\nu\rho]},\partial_\rho K^{\mu\nu\rho}\right)$ nor on the spacetime derivatives of $F_{\bar{A}}$ defined in (4.9), but only on the undifferentiated scalar field φ. With the help of the relations (2.23)–(2.28), it can be shown that both $H_5(\delta|d)$ at pure ghost number zero and $H_5^{\mathrm{inv}}(\delta|d)$ are generated by the elements

$$
\begin{aligned}
(W)_{\mu\nu\rho\lambda\sigma} = {} & \frac{dW}{d\varphi}C^*_{\mu\nu\rho\lambda\sigma}+\frac{d^2W}{d\varphi^2}\left(H^*_{[\mu}C^*_{\nu\rho\lambda\sigma]}+C^*_{[\mu\nu}C^*_{\rho\lambda\sigma]}\right)+\\
&+\frac{d^3W}{d\varphi^3}\left(H^*_{[\mu}H^*_{\nu}C^*_{\rho\lambda\sigma]}+H^*_{[\mu}C^*_{\nu\rho}C^*_{\lambda\sigma]}\right)+\frac{d^4W}{d\varphi^4}H^*_{[\mu}H^*_{\nu}H^*_{\rho}C^*_{\lambda\sigma]}+\\
&+\frac{d^5W}{d\varphi^5}H^*_{\mu}H^*_{\nu}H^*_{\rho}H^*_{\lambda}H^*_{\sigma}\,, \qquad (4.16)
\end{aligned}
$$

where $W=W(\varphi)$ is an arbitrary, smooth function of the undifferentiated scalar field φ. Indeed, direct computation yields

$$
\delta\,(W)_{\mu\nu\rho\lambda\sigma}=-\partial_{[\mu}\,(W)_{\nu\rho\lambda\sigma]}\,,\qquad \mathrm{agh}\left((W)_{\nu\rho\lambda\sigma}\right)=4\,, \qquad (4.17)
$$

where we made the notation

$$
\begin{aligned}
(W)_{\mu\nu\rho\lambda} = {} & \frac{dW}{d\varphi}C^*_{\mu\nu\rho\lambda}+\frac{d^2W}{d\varphi^2}\left(H^*_{[\mu}C^*_{\nu\rho\lambda]}+C^*_{[\mu\nu}C^*_{\rho\lambda]}\right)+\\
&+\frac{d^3W}{d\varphi^3}H^*_{[\mu}H^*_{\nu}C^*_{\rho\lambda]}+\frac{d^4W}{d\varphi^4}H^*_{\mu}H^*_{\nu}H^*_{\rho}H^*_{\lambda}\,. \qquad (4.18)
\end{aligned}
$$

Using again the actions of δ on the BRST generators, it can be proved that both $H_4(\delta|d)$ at pure ghost number zero and $H_4^{\mathrm{inv}}(\delta|d)$ are spanned by the elements $(W)_{\mu\nu\rho\lambda}$ given in (4.18) and by the undifferentiated antifields $\eta^*_{\mu\nu\rho\lambda\sigma}$ [the second definition in (2.28)]. Related to $(W)_{\mu\nu\rho\lambda}$, we have that

$$
\delta\,(W)_{\mu\nu\rho\lambda}=\partial_{[\mu}\,(W)_{\nu\rho\lambda]}\,,\qquad \mathrm{agh}\left((W)_{\nu\rho\lambda}\right)=3\,, \qquad (4.19)
$$

where we employed the convention

$$
(W)_{\mu\nu\rho}=\frac{dW}{d\varphi}C^*_{\mu\nu\rho}+\frac{d^2W}{d\varphi^2}H^*_{[\mu}C^*_{\nu\rho]}+\frac{d^3W}{d\varphi^3}H^*_{\mu}H^*_{\nu}H^*_{\rho}\,. \qquad (4.20)
$$

On account of the same arguments, it can be shown that the generators of the spaces $H_3(\delta|d)$ at pure ghost number zero and $H_3^{\mathrm{inv}}(\delta|d)$ are exactly $(W)_{\mu\nu\rho}$ expressed by (4.20), as well the undifferentiated antifields $\eta^*_{\mu\nu\rho\lambda}$, $\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}$, and C^* [see the formula (2.27)]. For the first element, straightforward calculations produce

$$
\delta\,(W)_{\mu\nu\rho}=-\partial_{[\mu}\,(W)_{\nu\rho]}\,,\qquad \mathrm{agh}\left((W)_{\nu\rho}\right)=2\,, \qquad (4.21)
$$

where we used the notation

$$
(W)_{\mu\nu}=\frac{dW}{d\varphi}C^*_{\mu\nu}+\frac{d^2W}{d\varphi^2}H^*_{\mu}H^*_{\nu}\,. \qquad (4.22)
$$

Finally, it can be proved that the spaces $H_2(\delta|d)$ at pure ghost number zero and $H_2^{\mathrm{inv}}(\delta|d)$ are spanned by $(W)_{\mu\nu}$ defined in (4.22) and by the undifferentiated antifields $\eta^*_{\mu\nu\rho}$, $\mathcal{G}^*_{\mu\nu\rho\lambda}$, C^*_μ, and η^* [see the first and last relations in (2.25), as well as the first two definitions from (2.26)]. Concerning $(W)_{\mu\nu}$, simple computation leads to

$$\delta(W)_{\mu\nu} = \partial_{[\mu}(W)_{\nu]}\,, \qquad \mathrm{agh}\left((W)_\nu\right) = 1\,, \tag{4.23}$$

with

$$(W)_\mu = \frac{dW}{d\varphi} H^*_\mu\,. \tag{4.24}$$

In contrast to the spaces $\left(H_J(\delta|d)\right)_{J\geq 2}$ and $\left(H_J^{\mathrm{inv}}(\delta|d)\right)_{J\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ at pure ghost number zero, that is related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free. Fortunately, it will not be needed in the sequel.

The previous results on $H(\delta|d)$ and $H^{\mathrm{inv}}(\delta|d)$ in strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. More precisely, we can successively eliminate all the pieces of antighost number strictly greater that five from the non-integrated density of the first-order deformation by adding solely trivial terms, so we can take, without loss of nontrivial objects, the condition $I \leq 5$ in the decomposition (4.3). The proof of this statement is contained in the appendix C. In addition, the last representative is of the form (4.11), where the invariant polynomial is necessarily a nontrivial object from $H_5^{\mathrm{inv}}(\delta|d)$.

4.2 First-order deformation

Using the results stated in the previous subsection, we can assume that the first-order deformation stops at antighost number five ($I = 5$)

$$a = a_0 + a_1 + a_2 + a_3 + a_4 + a_5\,, \tag{4.25}$$

where a_5 is of the form (4.11), with α_5 from $H_5^{\mathrm{inv}}(\delta|d)$ [elements of the form (4.16), generated by arbitrarily smooth functions, exclusively depending on the undifferentiated scalar field φ] and $e^5(\eta^{\bar{\Upsilon}})$ denoting the elements with pure ghost number five of a basis in the space of the polynomials in the ghosts $\eta^{\bar{\Upsilon}}$

$$e^5 : \left(\eta C^{\mu\nu\rho\lambda\sigma}, C\eta^{\mu\nu\rho\lambda\sigma}, \eta C\mathcal{G}^{\mu\nu\rho\lambda\sigma}, \eta\mathcal{G}^{\mu\nu\rho\lambda\sigma}\mathcal{G}^{\mu'\nu'\rho'\lambda'\sigma'}, \eta CC, \eta^{\mu\nu\rho\lambda\sigma}\mathcal{G}^{\mu'\nu'\rho'\lambda'\sigma'}\right). \tag{4.26}$$

In order to couple (4.16) to the last three elements from (4.26) like in (4.11) we need some completely antisymmetric constants, which, by covariance arguments, can only be proportional with the completely antisymmetric five-dimensional symbol, $\varepsilon_{\mu\nu\rho\lambda\sigma}$. Thus, the most general (manifestly covariant) form of the last representative from the expansion (4.25) is given by

$$\begin{aligned} a_5 = {} & (W_1)_{\mu\nu\rho\lambda\sigma}\,\eta C^{\mu\nu\rho\lambda\sigma} + (W_2)_{\mu\nu\rho\lambda\sigma}\,C\eta^{\mu\nu\rho\lambda\sigma} + (W_3)_{\mu\nu\rho\lambda\sigma}\,\eta C\mathcal{G}^{\mu\nu\rho\lambda\sigma} - \\ & -\epsilon^{\alpha\beta\gamma\delta\varepsilon}\left((W_4)_{\mu\nu\rho\lambda\sigma}\,\eta\mathcal{G}^{\mu\nu\rho\lambda\sigma}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} - \frac{1}{5!}(W_5)_{\alpha\beta\gamma\delta\varepsilon}\,\eta CC + (W_6)_{\mu\nu\rho\lambda\sigma}\,\eta^{\mu\nu\rho\lambda\sigma}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\right), \end{aligned} \tag{4.27}$$

where each of the elements $\left((W_k)_{\mu\nu\rho\lambda\sigma}\right)_{k=\overline{1,6}}$ is expressed like in (4.16), being generated by an arbitrary smooth function of the undifferentiated scalar field, $W_k(\varphi)$.

Inserting (4.27) into the equation (4.5) for $I = 5$ and using the definitions (2.22)–(2.34), after some computation we obtain the piece with antighost number equal to four from the first-order deformation like

$$
\begin{aligned}
a_4 = {} & (W_1)_{\mu\nu\rho\lambda}\left(5A_\sigma C^{\mu\nu\rho\lambda\sigma} - \eta C^{\mu\nu\rho\lambda}\right) - (W_2)_{\mu\nu\rho\lambda}\left(5C_\sigma \eta^{\mu\nu\rho\lambda\sigma} + C\eta^{\mu\nu\rho\lambda}\right) + \\
& + (W_3)_{\mu\nu\rho\lambda}\left(4A_\sigma C\mathcal{G}^{\mu\nu\rho\lambda\sigma} + 4\eta C_\sigma \mathcal{G}^{\mu\nu\rho\lambda\sigma} - \eta C\mathcal{G}^{\mu\nu\rho\lambda}\right) + \\
& + \epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[-\left((W_4)_{[\mu\nu\rho\lambda} A_{\sigma]}\mathcal{G}^{\mu\nu\rho\lambda\sigma} - 2(W_4)_{\mu\nu\rho\lambda}\eta\mathcal{G}^{\mu\nu\rho\lambda}\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
& \quad + \frac{1}{5!}\left((W_5)_{[\alpha\beta\gamma\delta} A_{\varepsilon]}C - 2(W_5)_{[\alpha\beta\gamma\delta} C_{\varepsilon]}\eta\right)C + \\
& \quad + (W_6)_{\mu\nu\rho\lambda}\left(\eta^{\mu\nu\rho\lambda}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} - \mathcal{G}^{\mu\nu\rho\lambda}\eta_{\alpha\beta\gamma\delta\varepsilon}\right)\Big] + \\
& + 2\left((W_1)_{[\mu\nu\rho} B^*_{\lambda\sigma]} + (W_1)_{[\mu\nu}\eta^*_{\rho\lambda\sigma]} + \frac{dW_1}{d\varphi}H^*_{[\mu}\eta^*_{\nu\rho\lambda\sigma]} + W_1\eta^*_{\mu\nu\rho\lambda\sigma}\right)C^{\mu\nu\rho\lambda\sigma} + \\
& + 2\left((W_3)_{[\mu\nu\rho} B^*_{\lambda\sigma]} + (W_3)_{[\mu\nu}\eta^*_{\rho\lambda\sigma]} + \frac{dW_3}{d\varphi}H^*_{[\mu}\eta^*_{\nu\rho\lambda\sigma]} + W_3\eta^*_{\mu\nu\rho\lambda\sigma}\right)C\mathcal{G}^{\mu\nu\rho\lambda\sigma} - \\
& - 2\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[\Big((W_4)_{[\mu\nu\rho} B^*_{\lambda\sigma]} + (W_4)_{[\mu\nu}\eta^*_{\rho\lambda\sigma]} + \frac{dW_4}{d\varphi}H^*_{[\mu}\eta^*_{\nu\rho\lambda\sigma]} + W_4\eta^*_{\mu\nu\rho\lambda\sigma}\Big) \times \\
& \quad \times \mathcal{G}^{\mu\nu\rho\lambda\sigma}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} - \frac{1}{5!}\Big((W_5)_{[\alpha\beta\gamma} B^*_{\delta\varepsilon]} + (W_5)_{[\alpha\beta}\eta^*_{\gamma\delta\varepsilon]} + \\
& \qquad + \frac{dW_5}{d\varphi}H^*_{[\alpha}\eta^*_{\beta\gamma\delta\varepsilon]} + W_5\eta^*_{\alpha\beta\gamma\delta\varepsilon}\Big)CC\Big]. \qquad (4.28)
\end{aligned}
$$

Here and in the sequel $(W_k)_{\mu\nu\rho\lambda}$, $(W_k)_{\mu\nu\rho}$, and $(W_k)_{\mu\nu}$ are written like in (4.18), (4.20), and respectively (4.22), with $W(\varphi)$ replaced by the corresponding $W_k(\varphi)$.

From now on we will need the relations (4.19), (4.21), and (4.23). Substituting the solution (4.28) into the equation (4.6) for $i = 4$ and employing the same definitions like before, we derive the terms of antighost number three from the first-order deformation as

$$
\begin{aligned}
a_3 = {} & -(W_1)_{\mu\nu\rho}\left(4A_\lambda C^{\mu\nu\rho\lambda} - \eta C^{\mu\nu\rho}\right) + (W_2)_{\mu\nu\rho}\left(4C_\lambda\eta^{\mu\nu\rho\lambda} + C\eta^{\mu\nu\rho}\right) - \\
& - 2\left((W_1)_{[\mu\nu} B^*_{\rho\lambda]} + \frac{dW_1}{d\varphi}H^*_{[\mu}\eta^*_{\nu\rho\lambda]} + W_1\eta^*_{\mu\nu\rho\lambda}\right)C^{\mu\nu\rho\lambda} - (W_2)_{[\mu\nu\rho}\phi_{\lambda\sigma]}\eta^{\mu\nu\rho\lambda\sigma} + \\
& + \Big((W_3)_{[\mu\nu\rho} A_\lambda C_{\sigma]} + 2(W_3)_{[\mu\nu} B^*_{\rho\lambda} C_{\sigma]} + \\
& \quad + 2\frac{dW_3}{d\varphi}H^*_{[\mu}\eta^*_{\nu\rho\lambda}C_{\sigma]} + 2W_3\eta^*_{[\mu\nu\rho\lambda}C_{\sigma]} - (W_3)_{[\mu\nu\rho}\phi_{\lambda\sigma]}\eta\Big)\mathcal{G}^{\mu\nu\rho\lambda\sigma} - \\
& - \Big[\Big((W_3)_{[\mu\nu\rho} A_{\lambda]} + 2(W_3)_{[\mu\nu} B^*_{\rho\lambda]} + 2\frac{dW_3}{d\varphi}H^*_{[\mu}\eta^*_{\nu\rho\lambda]} + 2W_3\eta^*_{\mu\nu\rho\lambda}\Big)C - \\
& \quad - (W_3)_{[\mu\nu\rho} C_{\lambda]}\eta\Big]\mathcal{G}^{\mu\nu\rho\lambda} + (W_3)_{\mu\nu\rho}\eta C K^{\mu\nu\rho} + \\
& + \epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[4\Big(\frac{1}{2}(W_4)_{[\mu\nu\rho} A_{\lambda]} + (W_4)_{[\mu\nu} B^*_{\rho\lambda]} + \frac{dW_4}{d\varphi}H^*_{[\mu}\eta^*_{\nu\rho\lambda]} + W_4\eta^*_{\mu\nu\rho\lambda}\Big) \times
\end{aligned}
$$

$$
\begin{aligned}
&\times \mathcal{G}^{\mu\nu\rho\lambda}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \left(\frac{1}{3!}\left(W_4\right)_{\alpha\beta\gamma}\epsilon_{\delta\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\varepsilon\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'} - \right.\\
&\qquad\qquad\qquad \left. - 2\left(W_4\right)_{\mu\nu\rho}K^{\mu\nu\rho}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\right)\eta + \\
&+ \frac{4}{5!}\left(\frac{1}{2}\left(W_5\right)_{[\alpha\beta\gamma}A_\delta C_{\varepsilon]} + \left(W_5\right)_{[\alpha\beta}B^*_{\gamma\delta}C_{\varepsilon]} + \right.\\
&\qquad \left. + \frac{dW_5}{d\varphi}H^*_{[\alpha}\eta^*_{\beta\gamma\delta}C_{\varepsilon]} + W_5\eta^*_{[\alpha\beta\gamma\delta}C_{\varepsilon]}\right)C + \\
&+ \frac{2}{5!}\left(\left(W_5\right)_{[\alpha\beta\gamma}C_\delta C_{\varepsilon]} - \left(W_5\right)_{[\alpha\beta\gamma}\phi_{\delta\varepsilon]}C\right)\eta - \\
&- \left(W_6\right)_{\mu\nu\rho}\left(\eta^{\mu\nu\rho}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + K^{\mu\nu\rho}\eta_{\alpha\beta\gamma\delta\varepsilon} - \eta^{\mu\nu\rho\lambda}\sigma_{\lambda\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon}\right)\Big] - \\
-3\Big[&\left(\left(W_2\right)_{[\mu\nu}K^*_{\rho\lambda\sigma]} + \frac{dW_2}{d\varphi}H^*_{[\mu}\mathcal{G}^*_{\nu\rho\lambda\sigma]} + W_2\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}\right)\eta^{\mu\nu\rho\lambda\sigma} + \\
&+ \left(\left(W_3\right)_{[\mu\nu}K^*_{\rho\lambda\sigma]} + \frac{dW_3}{d\varphi}H^*_{[\mu}\mathcal{G}^*_{\nu\rho\lambda\sigma]} + W_3\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}\right)\eta\mathcal{G}^{\mu\nu\rho\lambda\sigma} + \\
&+ \left(\left(W_3\right)_{\mu\nu}\phi^{*\mu\nu} + \frac{dW_3}{d\varphi}H^*_\mu C^{*\mu} - W_3C^*\right)\eta C\Big] + \\
+\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[&6\left(\left(W_4\right)_{\mu\nu}\phi^{*\mu\nu} + \frac{dW_4}{d\varphi}H^*_\mu C^{*\mu} - W_4C^*\right)\eta\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} - \\
&- \frac{1}{20}\left(\left(W_5\right)_{[\alpha\beta}K^*_{\gamma\delta\varepsilon]} + \frac{dW_5}{d\varphi}H^*_{[\alpha}\mathcal{G}^*_{\beta\gamma\delta\varepsilon]} + W_5\mathcal{G}^*_{\alpha\beta\gamma\delta\varepsilon}\right)\eta C + \\
&+ 3\left(\left(W_6\right)_{\mu\nu}\phi^{*\mu\nu} + \frac{dW_6}{d\varphi}H^*_\mu C^{*\mu} - W_6C^*\right)\eta_{\alpha\beta\gamma\delta\varepsilon}\Big].
\end{aligned}
\tag{4.29}
$$

The component with the antighost number equal to two results as solution to the equation (4.6) for $i = 3$, by relying on the formula (4.29) and the definitions (2.22)–(2.34), and takes the form

$$
\begin{aligned}
a_2 = {} & \left(W_1\right)_{\mu\nu}\left(3A_\rho C^{\mu\nu\rho} - \eta C^{\mu\nu}\right) - \left(W_2\right)_{\mu\nu}\left(3C_\rho\eta^{\mu\nu\rho} + B^{\mu\nu}C\right) + \\
& +2\left(\frac{dW_1}{d\varphi}H^*_{[\mu}B^*_{\nu\rho]} + W_1\eta^*_{\mu\nu\rho}\right)C^{\mu\nu\rho} + \\
& + \left(\left(W_2\right)_{[\mu\nu}\phi_{\rho\lambda]} + 3\frac{dW_2}{d\varphi}H^*_{[\mu}K^*_{\nu\rho\lambda]} + 3W_2\mathcal{G}^*_{\mu\nu\rho\lambda}\right)\eta^{\mu\nu\rho\lambda} - \\
& - \Big[\left(W_3\right)_{[\mu\nu}A_\rho\phi_{\lambda\sigma]} + 2\frac{dW_3}{d\varphi}H^*_{[\mu}B^*_{\nu\rho}\phi_{\lambda\sigma]} + \\
& \qquad + 3\frac{dW_3}{d\varphi}H^*_{[\mu}K^*_{\nu\rho\lambda}A_{\sigma]} + W_3\left(2\eta^*_{[\mu\nu\rho}\phi_{\lambda\sigma]} + 3\mathcal{G}^*_{[\mu\nu\rho\lambda}A_{\sigma]}\right)\Big]\mathcal{G}^{\mu\nu\rho\lambda\sigma} - \\
& - \Big[\left(W_3\right)_{[\mu\nu}A_\rho C_{\lambda]} - \left(W_3\right)_{[\mu\nu}\phi_{\rho\lambda]}\eta + 2\frac{dW_3}{d\varphi}H^*_{[\mu}B^*_{\nu\rho}C_{\lambda]} - \\
& \qquad - 3\frac{dW_3}{d\varphi}H^*_{[\mu}K^*_{\nu\rho\lambda]}\eta + W_3\left(2\eta^*_{[\mu\nu\rho}C_{\lambda]} - 3\mathcal{G}^*_{\mu\nu\rho\lambda}\eta\right)\Big]\mathcal{G}^{\mu\nu\rho\lambda} + \\
& + \left(\left(W_3\right)_{[\mu\nu}A_{\rho]} + 2\frac{dW_3}{d\varphi}H^*_{[\mu}B^*_{\nu\rho]} + 2W_3\eta^*_{\mu\nu\rho}\right)K^{\mu\nu\rho}C +
\end{aligned}
$$

$$
\begin{aligned}
&+3\left(2\frac{dW_3}{d\varphi}H^*_\mu\phi^{*\mu\nu}-W_3C^{*\nu}\right)A_\nu C+\\
&+3\left((W_3)_{\mu\nu}K^{\mu\nu\rho}+2\frac{dW_3}{d\varphi}H^*_\mu\phi^{*\mu\rho}-W_3C^{*\rho}\right)\eta C_\rho+\\
&+\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Bigg[-2\left((W_4)_{[\mu\nu}A_{\rho]}+2\frac{dW_4}{d\varphi}H^*_{[\mu}B^*_{\nu\rho]}+2W_4\eta^*_{\mu\nu\rho}\right)K^{\mu\nu\rho}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}+\\
&\qquad+\frac{1}{3}\left(\frac{1}{2}(W_4)_{[\alpha\beta}A_{\gamma]}+\frac{dW_4}{d\varphi}H^*_{[\alpha}B^*_{\beta\gamma]}+W_4\eta^*_{\alpha\beta\gamma}\right)\times\\
&\qquad\times\epsilon_{\delta\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\varepsilon\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'}+\\
&\qquad+6\left((W_4)_{\mu\nu}K^{\mu\nu\rho}+2\frac{dW_4}{d\varphi}H^*_\mu\phi^{*\mu\rho}-W_4C^{*\rho}\right)\sigma_{\rho\alpha}\eta\mathcal{G}_{\beta\gamma\delta\varepsilon}-\\
&\qquad-6\left(\frac{dW_4}{d\varphi}H^*_{[\mu}A_{\nu]}\phi^{*\mu\nu}-W_4C^{*\mu}A_\mu\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}-\\
&\qquad-\frac{2}{5!}\left((W_5)_{[\alpha\beta}A_\gamma\phi_{\delta\varepsilon]}+2\frac{dW_5}{d\varphi}H^*_{[\alpha}B^*_{\beta\gamma}\phi_{\delta\varepsilon]}+2W_5\eta^*_{[\alpha\beta\gamma}\phi_{\delta\varepsilon]}\right)C-\\
&\qquad-\frac{1}{20}\left(\frac{dW_5}{d\varphi}H^*_{[\alpha}K^*_{\beta\gamma\delta}A_{\varepsilon]}+W_5\mathcal{G}^*_{[\alpha\beta\gamma\delta}A_{\varepsilon]}\right)C+\\
&\qquad+\frac{1}{20}\left(\frac{1}{3}(W_5)_{[\alpha\beta}\phi_{\gamma\delta}C_{\varepsilon]}+\frac{1}{2}\frac{dW_5}{d\varphi}H^*_{[\alpha}K^*_{\beta\gamma\delta}C_{\varepsilon]}+W_5\mathcal{G}^*_{[\alpha\beta\gamma\delta}C_{\varepsilon]}\right)\eta+\\
&\qquad+\frac{2}{5!}\left((W_5)_{[\alpha\beta}A_\gamma C_\delta C_{\varepsilon]}+2\frac{dW_5}{d\varphi}H^*_{[\alpha}B^*_{\beta\gamma}C_\delta C_{\varepsilon]}+2W_5\eta^*_{[\alpha\beta\gamma}C_\delta C_{\varepsilon]}\right)+\\
&\qquad+(W_6)_{\mu\nu}\left(B^{\mu\nu}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}+3\eta^{\mu\nu\rho}\sigma_{\rho\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon}\right)+\\
&\qquad+3\left((W_6)_{\mu\nu}K^{\mu\nu\rho}+2\frac{dW_6}{d\varphi}H^*_\mu\phi^{*\mu\rho}-W_6C^{*\rho}\right)\sigma_{\rho\alpha}\eta_{\beta\gamma\delta\varepsilon}\Bigg]-\\
&-2\left(\frac{dW_2}{d\varphi}H^*_\mu A^{*\mu}-W_2\eta^*\right)C-6W_3\left(\phi^{*\mu\nu}B^*_{\mu\nu}C+K^*_{[\mu\nu\rho}B^*_{\lambda\sigma]}\mathcal{G}^{\mu\nu\rho\lambda\sigma}\right)+\\
&+\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Bigg[12W_4\phi^{*\mu\nu}B^*_{\mu\nu}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}+\frac{1}{10}W_5B^*_{[\alpha\beta}K^*_{\gamma\delta\varepsilon]}C+\\
&\qquad+2\left(\frac{dW_6}{d\varphi}H^*_\mu A^{*\mu}-W_6\eta^*\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\Bigg].
\end{aligned}
\tag{4.30}
$$

Replacing now the expression (4.30) into the equation (4.6) for $i=2$ and recalling the same definitions like in the above, we obtain that the piece of antighost number one in the first-order deformation is written like

$$
\begin{aligned}
a_1 = &-\frac{dW_1}{d\varphi}H^*_\mu\left(2A_\nu C^{\mu\nu}-H^\mu\eta\right)+\frac{dW_2}{d\varphi}H^*_\mu\left(2B^{\mu\nu}C_\nu-3\phi_{\nu\rho}\eta^{\mu\nu\rho}\right)-\\
&-2W_1B^*_{\mu\nu}C^{\mu\nu}-W_2\left(3K^*_{\mu\nu\rho}\eta^{\mu\nu\rho}+2A^{*\mu}C_\mu\right)+\\
&+\left[\frac{dW_3}{d\varphi}H^*_{[\mu}A_\nu\phi_{\rho\lambda]}+W_3\left(2B^*_{[\mu\nu}\phi_{\rho\lambda]}+3K^*_{[\mu\nu\rho}A_{\lambda]}\right)\right]\mathcal{G}^{\mu\nu\rho\lambda}+\\
&+3\left[\frac{dW_3}{d\varphi}H^*_\mu\left(2A_\nu C_\rho-\phi_{\nu\rho}\eta\right)+W_3\left(2B^*_{\mu\nu}C_\rho-K^*_{\mu\nu\rho}\eta\right)\right]K^{\mu\nu\rho}-\\
&-3W_3\phi^*_{\mu\nu}\left(A^{[\mu}C^{\nu]}-\phi^{\mu\nu}\eta\right)+
\end{aligned}
$$

$$
\begin{aligned}
&+\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Bigg[6\left(\frac{dW_4}{d\varphi}H^*_{[\mu}A_{\nu]}+2W_4B^*_{\mu\nu}\right)K^{\mu\nu\rho}\sigma_{\rho\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon}+\\
&\qquad+\frac{1}{4}\frac{dW_4}{d\varphi}H^*_{\alpha}\epsilon_{\beta\gamma\mu\nu\rho}K^{\mu\nu\rho}\epsilon_{\delta\varepsilon\mu'\nu'\rho'}K^{\mu'\nu'\rho'}+\\
&\qquad+6W_4\left(\phi^*_{\alpha\beta}K_{\gamma\delta\varepsilon}\eta-2\phi^{*\mu\nu}A_\mu\sigma_{\nu\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon}\right)-\\
&\qquad-\frac{1}{5!}\left(2\frac{dW_5}{d\varphi}H^*_{[\alpha}A_\beta\phi_{\gamma\delta}C_{\varepsilon]}+4W_5B^*_{[\alpha\beta}\phi_{\gamma\delta}C_{\varepsilon]}+6W_5K^*_{[\alpha\beta\gamma}A_\delta C_{\varepsilon]}\right)+\\
&\qquad+\frac{1}{5!}\left(2\frac{dW_5}{d\varphi}H^*_{[\alpha}\phi_{\beta\gamma}\phi_{\delta\varepsilon]}+6W_5K^*_{[\alpha\beta\gamma}\phi_{\delta\varepsilon]}\right)\eta+\\
&\qquad+2\left(\frac{dW_6}{d\varphi}H^*_\mu B^{\mu\nu}-W_6A^{*\nu}\right)\sigma_{\nu\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon}-\\
&\qquad-3\left(\frac{dW_6}{d\varphi}H^{*\mu}K_{\mu\alpha\beta}-W_6\phi^*_{\alpha\beta}\right)\eta_{\gamma\delta\varepsilon}\Bigg]-W_1\varphi^*\eta\,.
\end{aligned}
\tag{4.31}
$$

In the last step we solve the equation (4.6) for $i=1$ with the help of the relation (4.31) and the definitions of γ acting on the BRST generators, whose solution reads as

$$
\begin{aligned}
a_0 = {}& W_1A_\mu H^\mu+W_2B_{\mu\nu}\phi^{\mu\nu}-W_3\phi_{[\mu\nu}A_{\rho]}K^{\mu\nu\rho}+\\
&+\epsilon^{\alpha\beta\gamma\delta\varepsilon}\left(\frac{1}{4}W_4A_\alpha\epsilon_{\beta\gamma\mu\nu\rho}K^{\mu\nu\rho}\epsilon_{\delta\varepsilon\mu'\nu'\rho'}K^{\mu'\nu'\rho'}+\right.\\
&\qquad\left.+\frac{1}{4}W_5A_\alpha\phi_{\beta\gamma}\phi_{\delta\varepsilon}+W_6B_{\alpha\beta}K_{\gamma\delta\varepsilon}\right)+\bar{M}(\varphi)\,,
\end{aligned}
\tag{4.32}
$$

and represents nothing but the interacting lagrangian at order one in the coupling constant. The solution $\bar{M}(\varphi)$ represents the general solution to the 'homogeneous' equation

$$
\gamma\bar{a}_0=\partial_\mu j_0^\mu\,, \tag{4.33}
$$

which is determined in appendix D. (The solutions to this 'homogeneous' equation come from $\bar{a}_1=0$, and hence they bring contributions only to the deformed lagrangian density at order one in the coupling constant.)

We emphasize that the solutions $(a_m)_{m=\overline{0,4}}$ obtained in the above also include the solutions corresponding to the associated 'homogeneous' equations $\gamma\bar{a}_m=0$. In order to simplify the exposition we avoided the discussion regarding the selection procedure of these solutions such as to comply with obtaining some consistent components of the first-order deformation at each value of the antighost number. It is however interesting to note that this procedure allows no new functions of the scalar fields beside $(W_k)_{k=\overline{1,6}}$ and $\bar{M}(\varphi)$ to enter $(a_m)_{m=\overline{0,4}}$. In consequence, we succeeded in finding the complete form of the non-integrated density of the first-order deformation of the solution to the master equation for the model under study (4.25), which reduces to a sum of terms with antighost numbers ranging from zero to five, namely, the right-hand sides of the formulas (4.27)–(4.32), and is parametrized in terms of seven arbitrary, smooth functions of the undifferentiated scalar field φ.

4.3 Higher-order deformations

Next, we investigate the equations responsible for higher-order deformations. The second-order deformation is governed by the equation (3.6). Making use of the first-order deformation derived in the previous subsection, after some computation we organize the second term in the left-hand side of (3.6) like

$$(S_1, S_1) = \int d^5x \left(Y^{(0)} X^{(0)} + \sum_{a=0}^{5} \sum_{i=1}^{8} \frac{d^a Y^{(i)}}{d\varphi^a} X_a^{(i)} \right), \tag{4.34}$$

where

$$Y^{(0)}(\varphi) = \frac{d\bar{M}(\varphi)}{d\varphi} W_1(\varphi), \tag{4.35}$$

$$Y^{(1)}(\varphi) = W_1(\varphi) W_2(\varphi), \tag{4.36}$$

$$Y^{(2)}(\varphi) = W_1(\varphi) \frac{dW_2(\varphi)}{d\varphi} - 3W_2(\varphi) W_3(\varphi) + 6W_5(\varphi) W_6(\varphi), \tag{4.37}$$

$$Y^{(3)}(\varphi) = W_2(\varphi) W_3(\varphi) + W_5(\varphi) W_6(\varphi), \tag{4.38}$$

$$Y^{(4)}(\varphi) = W_1(\varphi) \frac{dW_6(\varphi)}{d\varphi} + 3W_3(\varphi) W_6(\varphi) - 6W_2(\varphi) W_4(\varphi), \tag{4.39}$$

$$Y^{(5)}(\varphi) = W_1(\varphi) W_6(\varphi), \tag{4.40}$$

$$Y^{(6)}(\varphi) = W_2(\varphi) W_4(\varphi) + W_3(\varphi) W_6(\varphi), \tag{4.41}$$

$$Y^{(7)}(\varphi) = W_2(\varphi) W_5(\varphi), \tag{4.42}$$

$$Y^{(8)}(\varphi) = W_4(\varphi) W_6(\varphi), \tag{4.43}$$

while the remaining objects, namely, $X^{(0)}$ and $(X_a^{(i)})_{a=\overline{1,5},\, i=\overline{1,8}}$ can be found in the appendix E. On the one hand, $X^{(0)}$ and all $(X_a^{(i)})_{a=\overline{1,5},\, i=\overline{1,8}}$ are polynomials of ghost number one involving only the *undifferentiated* fields/ghosts and antifields. On the other hand, the equation (3.6) requires that (S_1, S_1) is s-exact. However, since none of the terms present in the right-hand side of (4.34) can be brought to such a form, the non-integrated density of (S_1, S_1) must vanish. This takes place if and only if the following equations are simultaneously obeyed

$$Y^{(k)}(\varphi) = 0, \ k = \overline{0,8}. \tag{4.44}$$

Using the above results, we can further take $S_2 = 0$, the remaining higher-order deformation equations being satisfied with the choice

$$S_k = 0, \qquad k > 2. \tag{4.45}$$

In this way the complete deformation of the solution to the master equation, consistent to all orders in the coupling constant, simply reduces to the sum between the 'free' solution (2.35) and the first-order deformation

$$S = \bar{S} + gS_1 = \bar{S} + g \int d^5x \left(\sum_{k=0}^{5} a_k \right), \tag{4.46}$$

where the components $(a_k)_{k=\overline{0,5}}$ are given in (4.27)–(4.32) and the functions $\bar{M}(\varphi)$ and $(W_k(\varphi))_{k=\overline{1,6}}$ are no longer arbitrary; they must satisfy the equations (4.44).

5. Lagrangian formulation of the deformed gauge theory

By virtue of the discussion from the end of section 2 on the significance of terms with various antighost numbers in the solution to the master equation, at this stage we can extract all the information on the gauge structure of the coupled model. The antifield-independent piece in (4.46) provides the expression of the overall lagrangian action of the interacting gauge theory

$$\begin{aligned}\tilde{S}\left[\varphi,H,A,B,\phi,K\right] = \int d^5x \Big[& H_\mu\left(\partial^\mu\varphi+gW_1A^\mu\right)+g\bar{M}\left(\varphi\right)+\\ &+\frac{1}{2}B^{\mu\nu}\left(\partial_{[\mu}A_{\nu]}+2gW_2\phi_{\mu\nu}\right)+\\ &+\frac{1}{3}K^{\mu\nu\rho}\left(\partial_{[\mu}\phi_{\nu\rho]}-3gW_3\phi_{[\mu\nu}A_{\rho]}\right)+\\ &+g\varepsilon^{\mu\nu\rho\lambda\sigma}\left(\frac{1}{4}W_4A_\mu\varepsilon_{\nu\rho\alpha\beta\gamma}K^{\alpha\beta\gamma}\varepsilon_{\lambda\sigma\alpha'\beta'\gamma'}K^{\alpha'\beta'\gamma'}+\right.\\ &\left.\left.+\frac{1}{4}W_5A_\mu\phi_{\nu\rho}\phi_{\lambda\sigma}+W_6B_{\mu\nu}K_{\rho\lambda\sigma}\right)\right], \end{aligned}\tag{5.1}$$

while from the components with antighost number one we conclude that it is invariant under the gauge transformations

$$\bar{\delta}_{\epsilon,\xi}A^\mu = \partial^\mu\epsilon-2gW_2\xi^\mu-2gW_6\varepsilon^{\mu\nu\rho\lambda\sigma}\xi_{\nu\rho\lambda\sigma}\,,\tag{5.2}$$

$$\begin{aligned}\bar{\delta}_{\epsilon,\xi}H^\mu =\; & 2D_\nu\epsilon^{\mu\nu}+g\left(\frac{dW_1}{d\varphi}H^\mu-3\frac{dW_3}{d\varphi}K^{\mu\nu\rho}\phi_{\nu\rho}\right)\epsilon-\\ &-3g\frac{dW_2}{d\varphi}\phi_{\nu\rho}\epsilon^{\mu\nu\rho}+2g\left(\frac{dW_2}{d\varphi}B^{\mu\nu}-3\frac{dW_3}{d\varphi}K^{\mu\nu\rho}A_\rho\right)\xi_\nu+\\ &+12g\frac{dW_3}{d\varphi}A_\nu\phi_{\rho\lambda}\xi^{\mu\nu\rho\lambda}+2g\frac{dW_6}{d\varphi}B^{\mu\nu}\varepsilon_{\nu\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}+\\ &+3gK^{\mu\nu\rho}\left(4\frac{dW_4}{d\varphi}A_\nu\varepsilon_{\rho\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}-\frac{dW_6}{d\varphi}\varepsilon_{\nu\rho\alpha\beta\gamma}\epsilon^{\alpha\beta\gamma}\right)+\\ &+g\varepsilon^{\mu\nu\rho\lambda\sigma}\left[\frac{1}{4}\frac{dW_4}{d\varphi}\varepsilon_{\nu\rho\alpha\beta\gamma}K^{\alpha\beta\gamma}\varepsilon_{\lambda\sigma\alpha'\beta'\gamma'}K^{\alpha'\beta'\gamma'}\epsilon-\right.\\ &\left.-\frac{dW_5}{d\varphi}\phi_{\nu\rho}\left(A_\lambda\xi_\sigma-\frac{1}{4}\phi_{\lambda\sigma}\epsilon\right)\right],\end{aligned}\tag{5.3}$$

$$\bar{\delta}_{\epsilon,\xi}\varphi = -gW_1\epsilon\,,\tag{5.4}$$

$$\begin{aligned}\bar{\delta}_{\epsilon,\xi}B^{\mu\nu} =\; & -3\partial_\rho\epsilon^{\mu\nu\rho}-2gW_1\epsilon^{\mu\nu}+6gW_3\left(2\phi_{\rho\lambda}\xi^{\mu\nu\rho\lambda}+K^{\mu\nu\rho}\xi_\rho\right)+\\ &+g\left(12W_4K^{\mu\nu\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}-W_5\varepsilon^{\mu\nu\rho\lambda\sigma}\phi_{\rho\lambda}\xi_\sigma\right),\end{aligned}\tag{5.5}$$

$$\begin{aligned}\bar{\delta}_{\epsilon,\xi}\phi_{\mu\nu} =\; & D^{(-)}_{[\mu}\xi_{\nu]}+3g\left(W_3\phi_{\mu\nu}\epsilon-2W_4A_{[\mu}\varepsilon_{\nu]\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}\right)+\\ &+3g\varepsilon_{\mu\nu\rho\lambda\sigma}\left(2W_4K^{\rho\lambda\sigma}\epsilon+W_6\epsilon^{\rho\lambda\sigma}\right),\end{aligned}\tag{5.6}$$

$$\begin{aligned}\bar{\delta}_{\epsilon,\xi}K^{\mu\nu\rho} =\; & 4D^{(+)}_\lambda\xi^{\mu\nu\rho\lambda}-3g\left(W_2\epsilon^{\mu\nu\rho}+W_3K^{\mu\nu\rho}\epsilon\right)-\\ &-g\varepsilon^{\mu\nu\rho\lambda\sigma}W_5\left(A_\lambda\xi_\sigma-\frac{1}{2}\phi_{\lambda\sigma}\epsilon\right),\end{aligned}\tag{5.7}$$

where we employed the notations

$$D_\nu = \partial_\nu - g\frac{dW_1}{d\varphi}A_\nu\,, \qquad D_\nu^{(\pm)} = \partial_\nu \pm 3gW_3A_\nu\,. \tag{5.8}$$

The commutators among the deformed gauge transformations, as well as the accompanying reducibility relations, result from the analysis of the structure of terms with antighost numbers greater than one in (4.46) and are listed in the appendices F and respectively G.

However, the functions $(W_k)_{k=\overline{1,6}}$ and $\bar{M}(\varphi)$ are no longer arbitrary smooth functions of the undifferentiated scalar field. They are required to fulfill the equations (4.44) in order to ensure the consistency of the deformed solution to the master equation to all orders in the coupling constant. Let us analyze now the solutions to the system (4.44). It it easy to see that (4.44) is equivalent with the equations

$$\frac{d\bar{M}(\varphi)}{d\varphi}W_1(\varphi) = 0\,, \tag{5.9}$$

$$W_1(\varphi)W_2(\varphi) = 0\,, \tag{5.10}$$

$$W_1(\varphi)\frac{dW_2(\varphi)}{d\varphi} - 9W_2(\varphi)W_3(\varphi) = 0\,, \tag{5.11}$$

$$W_2(\varphi)W_3(\varphi) + W_5(\varphi)W_6(\varphi) = 0\,, \tag{5.12}$$

$$W_1(\varphi)\frac{dW_6(\varphi)}{d\varphi} + 9W_3(\varphi)W_6(\varphi) = 0\,, \tag{5.13}$$

$$W_1(\varphi)W_6(\varphi) = 0\,, \tag{5.14}$$

$$W_2(\varphi)W_4(\varphi) + W_3(\varphi)W_6(\varphi) = 0\,, \tag{5.15}$$

$$W_2(\varphi)W_5(\varphi) = 0\,, \tag{5.16}$$

$$W_4(\varphi)W_6(\varphi) = 0\,. \tag{5.17}$$

There are three different types of solutions to (5.9)–(5.17). The first type is described by the choice

$$W_1(\varphi) = W_3(\varphi) = W_4(\varphi) = W_5(\varphi) = 0\,, \tag{5.18}$$

with $\bar{M}(\varphi)$, $W_2(\varphi)$, and $W_6(\varphi)$ arbitrary smooth functions of the undifferentiated scalar field. The second kind of solutions is pictured by the pick

$$\bar{M}(\varphi) = W_2(\varphi) = W_6(\varphi) = 0\,, \tag{5.19}$$

with $W_1(\varphi)$, $W_3(\varphi)$, $W_4(\varphi)$, and $W_5(\varphi)$ arbitrary smooth functions of φ. Finally, the third sort of solutions is parametrized by

$$W_1(\varphi) = W_2(\varphi) = W_6(\varphi) = 0\,, \tag{5.20}$$

while $\bar{M}(\varphi)$, $W_3(\varphi)$, $W_4(\varphi)$, and $W_5(\varphi)$ remain arbitrary smooth functions of the undifferentiated scalar field. If we particularize the general results on the lagrangian formulation of the interacting BF model contained in this section, and also the formulas related to the commutators among the deformed gauge transformations and to the accompanying reducibility relations contained in the appendices F and G to the above solutions, we obtain

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three interacting theories that are in a way complementary to each other. More precisely, the first solution produces a deformed interacting BF theory with an open gauge algebra that closes on-shell in an abelian way, but on-shell first- and second-order reducibility relations. The second one leads to a coupled topological BF model displaying an open gauge algebra, which closes on-shell in a non-abelian manner, and on-shell reducibility relations for all the three levels. The last case yields an interacting BF model with an open gauge algebra, on-shell first-order reducibility relations, but off-shell second- and third-order reducibility relations (the second- and third-order reducibility functions are not modified by the deformation procedure).

6. Conclusion

In conclusion, in this paper we have generated the consistent lagrangian interactions in five spacetime dimensions that can be introduced among one scalar field, two types of one-forms, two sorts of two-forms, and one three-form, pictured in the free limit by an abelian topological field theory of BF-type, with abelian gauge transformations, which are off-shell, third-order reducible. Our treatment is mainly based on the lagrangian BRST deformation procedure, that relies on the construction of the consistent deformations of the solution to the master equation with the help of some cohomological techniques. The couplings are obtained under the hypotheses of smoothness, locality, (background) Lorentz invariance, Poincaré invariance, and the preservation of the number of derivatives on each field. As a result, we obtain three sorts of coupled models that are in a way complementary to each other. All of them underlies a deformed interacting BF theory with an open gauge algebra, which only closes on-shell, where on-shell means on the stationary surface of field equations for the coupled model. However, for the first situation it closes according to an abelian algebra, while for the second and third model it produces on-shell a non-abelian algebra. Related to the reducibility relations, we remark that the first model outputs on-shell first- and second-order reducibility relations, but off-shell third-order reducibility, the second describes a coupled topological BF model displaying on-shell reducibility relations to all the three levels, while the third coupled theory exhibits on-shell first-order reducibility relations, but off-shell second- and third-order redundancy relations.

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A. Cohomology of γ and related matters

In this section we study the first ingredient implied in the local BRST cohomology $H(s|d)$, namely, the cohomology algebra of the exterior longitudinal derivative $H(\gamma)$. Let a be an

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element of $H(\gamma)$ with definite pure ghost number, antighost number and form degree (deg)

$$\gamma a = 0\,, \qquad \mathrm{pgh}\,(a) = l \geq 0\,, \qquad \mathrm{agh}\,(a) = k \geq 0\,, \qquad \deg\,(a) = p \leq 5\,. \tag{A.1}$$

Extending the analysis realized in section 4.1 for non-integrated densities (0-forms) to objects that may have nonvanishing form degrees, we can state that the general, local solution to the equation (A.1) (up to trivial, γ-exact contributions) is of the type

$$a = \sum_J \alpha_J\left(\left[\chi^*_\Delta\right], \left[F_A\right]\right) e^J\left(\eta^\Upsilon\right), \tag{A.2}$$

where F_A denotes the set of γ-closed (gauge-invariant) quantities that can be constructed out of the original fields

$$F_A = \left\{\varphi, \partial^{[\mu}A^{\nu]}, \partial_\mu H^\mu, \partial^{[\mu}\tilde{B}^{\nu\rho\lambda]}, \partial_{[\mu}\phi_{\nu\rho]}, \partial^{[\mu}\tilde{K}^{\nu\rho]}\right\}, \tag{A.3}$$

and χ^*_Δ is explained in (2.19). The notation $e^J\left(\eta^\Upsilon\right)$ in (A.2) signifies here the elements of pure ghost number equal to l of a basis in the space of polynomials in the undifferentiated ghosts

$$\eta^\Upsilon = \left(\eta, C, \widetilde{\mathcal{G}}, \tilde{\eta}, \tilde{C}\right), \tag{A.4}$$

so they have the properties

$$\mathrm{pgh}\left(e^J\right) = l > 0\,, \qquad \mathrm{agh}\left(e^J\right) = 0\,, \qquad \deg\left(e^J\right) = 0\,. \tag{A.5}$$

By contrast to section 4.1, here we work with slightly modified quantities F_A and η^Υ instead of (4.9) and (4.10), such as to include tilde quantities, defined like the Hodge duals of the untilded ones

$$\overset{\sim}{\Psi}{}^{\mu_1\mu_2\ldots\mu_k} = \frac{1}{(5-k)!}\varepsilon^{\mu_1\ldots\mu_k\nu_1\ldots\nu_{5-k}}\Psi_{\nu_1\ldots\nu_{5-k}}\,. \tag{A.6}$$

In fact, (A.2) is nothing but the analogue of (4.11) in form degree p and written in terms of the newly defined ghosts and gauge-invariant quantities. Here, the objects α_J [obviously nontrivial in $H^0(\gamma)$] are p-forms and were taken to have a finite antighost number and a bounded number of derivatives, so they are local p-forms with coefficients that are polynomials in the antifields χ^*_Δ, in the quantities F_A (excluding the undifferentiated scalar field φ), and also in their spacetime derivatives (including the derivatives of φ). However, α_J may contain infinite, formal series in the undifferentiated scalar field φ. Due to their γ-closeness, $\gamma\alpha_I = 0$, and to their (partial) polynomial character, α_J will be called 'invariant polynomials'. In agreement with (A.1), they display the properties

$$\mathrm{pgh}\,(\alpha_J) = 0\,, \qquad \mathrm{agh}\,(\alpha_J) = k \geq 0\,, \qquad \deg\,(\alpha_J) = p \leq 5\,. \tag{A.7}$$

In antighost number zero the invariant polynomials are local p-forms with coefficients that are polynomials in F_A (excluding the undifferentiated scalar field φ) and also in their spacetime derivatives (including the derivatives of φ), with coefficients that may be infinite, formal series in the undifferentiated scalar field φ.

In order to establish that just $H(\gamma)$ is required at the computation of $H(s|d)$, and not the local cohomology of γ, we need the cohomology of the exterior spacetime differential d in the space of invariant polynomials, as well as other interesting properties, which are addressed below.

Theorem 1. *The cohomology of d in form degree strictly less than 5 is trivial in the space of invariant polynomials with strictly positive antighost number. This means that the conditions*

$$\gamma\alpha = 0\,, \qquad d\alpha = 0\,, \qquad agh\,(\alpha) > 0\,, \qquad \deg(\alpha) < 5\,, \qquad \alpha = \alpha\left([\chi^*_\Delta]\,, [F_A]\right), \quad \text{(A.8)}$$

imply

$$\alpha = d\beta\,, \tag{A.9}$$

for some invariant polynomial $\beta\left([\chi^*_\Delta]\,, [F_A]\right)$.

Proof. In order to prove the theorem, we decompose d like

$$d = d_0 + d_1\,, \tag{A.10}$$

where d_1 acts solely on the antifields χ^*_Δ and their derivatives, while d_0 acts exclusively on the γ-invariant objects F_A and on their derivatives

$$d_0 = \partial^0_{\mu_1} dx^{\mu_1}\,, \qquad d_1 = \partial^1_{\mu_1} dx^{\mu_1}\,, \tag{A.11}$$

with

$$\partial^0_{\mu_1} = F_{A,\mu_1}\frac{\partial}{\partial F_A} + F_{A,\mu_1\mu_2}\frac{\partial}{\partial F_{A,\mu_2}} + \cdots\,, \tag{A.12}$$

$$\partial^1_{\mu_1} = \chi^*_{\Delta,\mu_1}\frac{\partial^L}{\partial \chi^*_\Delta} + \chi^*_{\Delta,\mu_1\mu_2}\frac{\partial^L}{\partial \chi^*_{\Delta,\mu_2}} + \cdots\,. \tag{A.13}$$

We used the common convention $f_{,\mu_1} \equiv \partial f/\partial x^{\mu_1}$. Obviously, $d^2 = 0$ on invariant polynomials is equivalent with the nilpotency and anticommutation of its components acting on invariant polynomials

$$d_0^2 = 0 = d_1^2\,, \qquad d_0 d_1 + d_1 d_0 = 0\,. \tag{A.14}$$

The action of d_0 on a given invariant polynomial with say l derivatives of F_A and j derivatives of χ^*_Δ results in an invariant polynomial with $(l+1)$ derivatives of F_A and j derivatives of χ^*_Δ, while the action of d_1 on the same object leads to an invariant polynomial with l derivatives of F_A and $(j+1)$ derivatives of χ^*_Δ. In particular, d_0 gives zero when acting on an invariant polynomial that does not involve any of the objects F_A or of their derivatives, and the same is valid with respect to d_1 acting on an invariant polynomial that does not depend on any of the antifields χ^*_Δ or on their derivatives. With the help of the relations (A.12)–(A.13) we observe that

$$\mathrm{agh}\,(d_0) = \mathrm{agh}\,(d_1) = \mathrm{agh}\,(d) = 0\,, \tag{A.15}$$

such that neither of them changes the antighost number of the objects on which any of them acts.

For convenience, the antifields χ^*_Δ will be called "foreground" fields, and the γ-invariant objects F_A will be named "background" fields. So, d_0 acts just on the background fields and their derivatives, while d_1 acts solely on the foreground fields and their derivatives.

According to the proposition from page 363 in [42], we have that the entire cohomology of d_1 in form degree strictly less than 5 is trivial in the space of invariant polynomials with strictly positive antighost number. This means that

$$\alpha = \alpha\left(\left[\chi_\Delta^*\right], \left[F_A\right]\right), \qquad \mathrm{agh}\,(\alpha) = k > 0\,, \qquad \deg\,(\alpha) = p < 5\,, \qquad d_1\alpha = 0\,, \tag{A.16}$$

implies that

$$\alpha = d_1\beta, \tag{A.17}$$

with

$$\beta = \beta\left(\left[\chi_\Delta^*\right], \left[F_A\right]\right), \qquad \mathrm{agh}\,(\beta) = k > 0\,, \qquad \deg\,(\beta) = p - 1\,. \tag{A.18}$$

In particular, we have that if an invariant polynomial (of form degree $p < 5$ and with strictly positive antighost number) depending only on the undifferentiated antifields is d_1-closed, then it vanishes

$$\left(\bar{\alpha} = \bar{\alpha}\left(\chi_\Delta^*, \left[F_A\right]\right), \qquad \mathrm{agh}\,(\bar{\alpha}) > 0\,, \qquad \deg\,(\bar{\alpha}) = p < 5\,, \qquad d_1\bar{\alpha} = 0\right) \quad \Rightarrow \quad \bar{\alpha} = 0\,. \tag{A.19}$$

Just d_0 has nontrivial cohomology. For instance, any form exclusively depending on the antifields and their derivatives is d_0-closed, but it is clearly not d_0-exact.

Next, assume that α is a homogeneous form of degree $p < 5$ and antighost number $k > 0$ that satisfies the conditions (A.8). We decompose α according to the number of derivatives of the antifields

$$\alpha = \overset{(0)}{\alpha} + \overset{(1)}{\alpha} + \cdots + \overset{(s)}{\alpha}\,, \qquad \mathrm{agh}\left(\overset{(i)}{\alpha}\right) = k > 0\,, \qquad \deg\left(\overset{(i)}{\alpha}\right) = p < 5\,, \tag{A.20}$$

where $\overset{(i)}{\alpha}$ signifies the component from α with i derivatives of the antifields. (The decomposition contains a finite number of terms since α is by assumption local.) As α is an invariant polynomial of form degree $p < 5$ and strictly positive antighost number, each component $\left(\overset{(i)}{\alpha}\right)_{0\leq i\leq s}$ is an invariant polynomial with the same form degree and strictly positive antighost number. The proof of the theorem is realized in $(s+1)$ steps.

Step 1. Taking into account the splitting (A.10), the projection of the equation

$$d\alpha = 0 \tag{A.21}$$

on the maximum number of derivatives of the antifields $(s+1)$ produces

$$d_1 \overset{(s)}{\alpha} = 0\,, \tag{A.22}$$

and hence the triviality of the cohomology of d_1 ensures that

$$\overset{(s)}{\alpha} = d_1 \overset{(s-1)}{\beta}\,, \qquad \mathrm{agh}\left(\overset{(s-1)}{\beta}\right) = k > 0\,, \qquad \deg\left(\overset{(s-1)}{\beta}\right) = p - 1\,, \tag{A.23}$$

where $\overset{(s-1)}{\beta}$ is an invariant polynomial of form degree $(p-1)$, with strictly positive antighost number and containing only $(s-1)$ derivatives of the antifields. If we introduce the p-form

$$\alpha_1 = \alpha - d \overset{(s-1)}{\beta}\,, \tag{A.24}$$

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then the equation (A.21) together with the nilpotency of d further yield

$$d\alpha_1 = 0\,. \tag{A.25}$$

It is by construction an invariant polynomial of form degree p and of strictly positive antighost number and, most important, the maximum number of derivatives of the antifields from α_1 is equal to $(s-1)$. Indeed, if we replace (A.23) in (A.20) and then in (A.24), we get that

$$\alpha_1 = \overset{(0)}{\alpha} + \overset{(1)}{\alpha} + \cdots + \overset{(s-2)}{\alpha} + \overset{(s-1)}{\alpha} - d_0 \overset{(s-1)}{\beta}\,. \tag{A.26}$$

Then, the maximum number of derivatives of the antifields from the first s terms in the right-hand side of (A.26) is contained in $\overset{(s-1)}{\alpha}$, being equal to $(s-1)$, while $d_0 \overset{(s-1)}{\beta}$ has the same number of derivatives of the antifields like $\overset{(s-1)}{\beta}$, which is again $(s-1)$.

Step 2. If we project now the equation (A.25) on the maximum number of derivatives of the antifields (s), we infer that

$$d_1 \left(\overset{(s-1)}{\alpha} - d_0 \overset{(s-1)}{\beta} \right) = 0\,, \tag{A.27}$$

with $\overset{(s-1)}{\alpha} - d_0 \overset{(s-1)}{\beta}$ an invariant polynomial of form degree p and of strictly positive antighost number. Using again the triviality of the cohomology of d_1, we deduce that

$$\overset{(s-1)}{\alpha} - d_0 \overset{(s-1)}{\beta} = d_1 \overset{(s-2)}{\beta}\,, \qquad \mathrm{agh}\left(\overset{(s-2)}{\beta}\right) = k > 0\,, \qquad \deg\left(\overset{(s-2)}{\beta}\right) = p-1\,, \tag{A.28}$$

where $\overset{(s-2)}{\beta}$ is an invariant polynomial of form degree $(p-1)$, with strictly positive antighost number and containing only $(s-2)$ derivatives of the antifields. At this stage, we define the p-form

$$\alpha_2 = \alpha - d\left(\overset{(s-1)}{\beta} + \overset{(s-2)}{\beta} \right). \tag{A.29}$$

The equation (A.21) together with the nilpotency of d further yield

$$d\alpha_2 = 0\,. \tag{A.30}$$

Clearly, α_2 is an invariant polynomial of form degree p and of strictly positive antighost number. It is essential to remark that the maximum number of derivatives of the antifields from α_2 is equal to $(s-2)$. This results by inserting (A.23) and (A.28) in (A.20) and consequently in (A.29), which then gives

$$\alpha_2 = \overset{(0)}{\alpha} + \overset{(1)}{\alpha} + \cdots + \overset{(s-3)}{\alpha} + \overset{(s-2)}{\alpha} - d_0 \overset{(s-2)}{\beta}\,. \tag{A.31}$$

etc.

Step s. Proceeding in the same manner, at the s-th step we obtain an invariant polynomial of form degree p and with strictly positive antighost number, which contains only the undifferentiated antifields

$$\alpha_s = \alpha - d\left(\overset{(s-1)}{\beta} + \cdots + \overset{(0)}{\beta}\right) = \overset{(0)}{\alpha} - d_0 \overset{(0)}{\beta}\,, \tag{A.32}$$

$$\mathrm{agh}\left(\overset{(j)}{\beta}\right) = k > 0\,, \qquad \deg\left(\overset{(j)}{\beta}\right) = p - 1\,, \qquad 0 \leq j \leq s-1\,. \tag{A.33}$$

[All $\left(\overset{(j)}{\beta}\right)$ $0 \leq j \leq s-1$ are invariant polynomials.] The equation (A.21) and the nilpotency of d lead to the equation

$$d\alpha_s = 0\,. \tag{A.34}$$

Step $(s+1)$. The projection of (A.34) on the maximum number of derivatives of the antifields (one) is

$$d_1\left(\overset{(0)}{\alpha} - d_0 \overset{(0)}{\beta}\right) = 0\,, \qquad \mathrm{agh}\left(\overset{(0)}{\alpha} - d_0 \overset{(0)}{\beta}\right) = k > 0\,. \tag{A.35}$$

Taking into account the relations (A.35) and (A.19) (with $\bar{\alpha}$ replaced by $\overset{(0)}{\alpha} - d_0 \overset{(0)}{\beta}$) we get that

$$\overset{(0)}{\alpha} - d_0 \overset{(0)}{\beta} = 0\,, \tag{A.36}$$

which substituted in (A.32) finally allows us to write that

$$\alpha = d\beta\,, \tag{A.37}$$

with

$$\beta = \left(\overset{(s-1)}{\beta} + \cdots + \overset{(0)}{\beta}\right), \qquad \mathrm{agh}\,(\beta) = k > 0\,, \qquad \deg\,(\beta) = p - 1\,, \tag{A.38}$$

and this proves the theorem since β is an invariant polynomial of form degree $(p-1)$ and with strictly positive antighost number. $\square$

In form degree 5 the Theorem 1 is replaced with: let $\alpha = \rho dx^0 \wedge \cdots \wedge dx^4$ be a d-exact invariant polynomial of form degree 5 and of strictly positive antighost number, $\mathrm{agh}\,(\alpha) = k > 0$, $\deg\,(\alpha) = 5$, $\alpha = d\beta$. Then, one can take the 4-form β to be an invariant polynomial (of antighost number k). In dual notations, this means that if ρ with $\mathrm{agh}\,(\rho) = k > 0$ is an invariant polynomial whose Euler-Lagrange derivatives are all vanishing, $\rho = \partial_\mu j^\mu$, then j^μ can be taken to be also invariant. Theorem 1 can be generalized as follows.

Theorem 2. *The cohomology of d computed in $H(\gamma)$ is trivial in form degree strictly less than 5 and in strictly positive antighost number*

$$H_p^{g,k}(d, H(\gamma)) = 0\,, \qquad k > 0\,, \qquad p < 5\,, \tag{A.39}$$

where p is the form degree, k is the antighost number and g is the ghost number.

Proof. An element a from $H_p^{g,k}(d, H(\gamma))$ is a p-form of definite ghost number g and antighost number k, pertaining to the cohomology of γ, which is d-closed modulo γ

$$\gamma a = 0,\ da = \gamma\mu\,, \qquad \mathrm{agh}\,(a) = k\,, \qquad \mathrm{gh}\,(a) = g\,, \qquad \deg\,(a) = p\,. \tag{A.40}$$

The theorem states that if a satisfies the conditions (A.40) with $p < 5$ and $k > 0$, then a is trivial in $H_p^{g,k}(d, H(\gamma))$

$$a = d\nu + \gamma\rho\,, \qquad \gamma\nu = 0\,, \tag{A.41}$$

where

$$\mathrm{agh}\,(\nu) = \mathrm{agh}\,(\rho) = k > 0\,, \qquad \mathrm{gh}\,(\nu) = g\,, \qquad \mathrm{gh}\,(\rho) = g - 1\,, \tag{A.42}$$

$$\deg\,(\nu) = p - 1\,, \qquad \deg\,(\rho) = p < 5\,. \tag{A.43}$$

Since $g = l' - k$, with l' the pure ghost number of a, and l' takes positive values $l' \geq 0$, it follows that g is restricted to fulfill the condition $g \geq -k$. Thus, if $g < -k$, then $a = 0$. The theorem is thus trivially obeyed for $g < -k$.

We consider a nontrivial element a from $H(\gamma)$ of form degree $p < 5$, of antighost number $k > 0$

$$\gamma a = 0,\ \mathrm{agh}\,(a) = k > 0\,, \qquad \deg\,(a) = p < 5\,. \tag{A.44}$$

In agreement with the previous results, a can be expressed, up to γ-exact contributions, like

$$a = \sum_J \alpha_J e^J\,. \tag{A.45}$$

We will use in extenso the following obvious properties

$$\gamma^2 = 0\,, \qquad d^2 = 0\,, \qquad \gamma d + d\gamma = 0\,, \qquad \mathrm{pgh}\,(d) = 0\,, \qquad \deg\,(\gamma) = 0\,, \tag{A.46}$$

$$\sum \alpha_J e^J = \gamma\,(\text{something}) \Leftrightarrow (\alpha_J = 0 \qquad \text{for all } J)\,, \tag{A.47}$$

$$d\alpha_J = \alpha'_J\,, \tag{A.48}$$

where

$$\mathrm{agh}\left(\alpha'_J\right) = \mathrm{agh}\,(\alpha_J)\,, \qquad \deg\left(\alpha'_J\right) = \deg\,(\alpha_J) + 1\,. \tag{A.49}$$

By applying the exterior spacetime differential on a of the form (A.45), we infer that

$$da = \pm \sum_J (d\alpha_J)\, e^J + \sum_J \alpha_J \left(de^J\right). \tag{A.50}$$

By means of the relations (2.30)–(2.34), we get

$$d\eta = \partial_\mu \eta dx^\mu = \gamma\,(-A_\mu dx^\mu)\,, \qquad dC = \partial_\mu C dx^\mu = \gamma\,(-C_\mu dx^\mu)\,, \tag{A.51}$$

$$d\,\tilde{\mathcal{G}} = \partial_\mu\,\tilde{\mathcal{G}}\,dx^\mu = \gamma\left(\frac{1}{5}\,\tilde{\mathcal{G}}_\mu\,dx^\mu\right), \tag{A.52}$$

$$d\tilde{\eta} = \partial_\mu \tilde{\eta} dx^\mu = \gamma\left(\frac{1}{5}\tilde{\eta}_\mu dx^\mu\right), \qquad d\tilde{C} = \partial_\mu \tilde{C} dx^\mu = \gamma\left(\frac{1}{5}\tilde{C}_\mu dx^\mu\right), \tag{A.53}$$

which allow us to write

$$de^J = \gamma \hat{e}^J \,, \tag{A.54}$$

where $\hat{e}^J$ depend in general on A_μ, C_μ, $\widetilde{\mathcal{G}}_\mu$, $\tilde{\eta}_\mu$, and $\tilde{C}_\mu$. Substituting (A.54) in (A.50), it follows that

$$da = \pm \sum_J (d\alpha_J)\, e^J + \gamma \left(\sum_J \alpha_J \hat{e}^J \right) . \tag{A.55}$$

Since a is a d-closed modulo γ p-form, the equations (A.40) and (A.55) yield

$$\pm \sum_J (d\alpha_J)\, e^J = \gamma \mu' \,. \tag{A.56}$$

With the help of the property (A.47), from (A.56) we arrive to

$$d\alpha_J = 0 \,. \tag{A.57}$$

Theorem 1 then implies that

$$\alpha_J = d\beta_J \,, \tag{A.58}$$

with β_J an invariant polynomial. Inserting α_J of the form (A.58) in (A.45), we obtain that

$$a = \sum_J d\beta_J e^J = \pm d \left(\sum_J \beta_J e^J \right) \mp \gamma \left(\sum_J \beta_J \hat{e}^J \right) , \tag{A.59}$$

which proves the theorem. □

Theorem 2 is one of the main tools needed for the computation of $H\,(s|d)$. In particular, it implies that there is no nontrivial descent for $H\,(\gamma|d)$ in strictly positive antighost number.

Corollary 1. *If a with*

$$agh\,(a) = k > 0 \,, \qquad gh\,(a) = g \geq -k \,, \qquad \deg\,(a) = p \leq 5 \,, \tag{A.60}$$

satisfies the equation

$$\gamma a + db = 0 \,, \tag{A.61}$$

where

$$agh\,(b) = k > 0 \,, \qquad gh\,(b) = g + 1 > -k \,, \qquad \deg\,(b) = p - 1 < 5 \,, \tag{A.62}$$

then one can always redefine a

$$a \rightarrow a' = a + d\nu \,, \tag{A.63}$$

so that

$$\gamma a' = 0 \,. \tag{A.64}$$

Proof. We construct the descent associated with the equation (A.61). Acting with γ on (A.61) and using the first and the third relations in (A.46), we find that

$$d\,(-\gamma b) = 0\,, \tag{A.65}$$

such that the triviality of the cohomology of d implies that

$$\gamma b + dc = 0\,, \tag{A.66}$$

where

$$\mathrm{agh}\,(c) = k > 0\,, \qquad \mathrm{gh}\,(c) = g + 2\,, \qquad \deg\,(c) = p - 2\,. \tag{A.67}$$

Going on in the same way, we get the next equation from the descent

$$\gamma c + de = 0\,, \tag{A.68}$$

with

$$\mathrm{agh}\,(e) = k > 0\,, \qquad \mathrm{gh}\,(e) = g + 3\,, \qquad \deg\,(e) = p - 3\,, \tag{A.69}$$

and so on. The descent stops after a finite number of steps with the last equations

$$\gamma t + du = 0\,, \tag{A.70}$$

$$\gamma u + dv = 0\,, \tag{A.71}$$

$$\gamma v = 0\,, \tag{A.72}$$

either because v is a zero-form or because we stopped at a higher form-degree with a γ-closed term. It is essential to remark that irrespective of the step at which the descent is cut, we have that

$$\mathrm{agh}\,(v) = k > 0\,, \qquad \mathrm{gh}\,(v) = g' > -k\,, \qquad \deg\,(v) = p' < 5\,. \tag{A.73}$$

[The earliest step where the descent may terminate is $v = b$ and, according to (A.62), we have that $\deg\,(b) = p - 1 < 5$ and $\mathrm{gh}\,(b) = g + 1 > -k$.]

The equations (A.71)–(A.72) together with the conditions (A.73) tell us that v belongs to $H_{p'}^{g',k}\,(d, H\,(\gamma))$ for $k > 0$, $p' < 5$ and $g' > -k$, so Theorem 2 guarantees that v is trivial in $H_{p'}^{g',k}\,(d, H\,(\gamma))$

$$v = d\nu' + \gamma\rho'\,, \qquad \gamma\nu' = 0\,, \tag{A.74}$$

which substituted in (A.71) allows us, due to the anticommutation between d and γ, to replace it with the equivalent equation

$$\gamma u' = 0\,, \tag{A.75}$$

where

$$u' = u - d\rho'\,. \tag{A.76}$$

In the meantime, (A.76) and the nilpotency of d induces that $du' = du$, such that the equation (A.70) becomes

$$\gamma t + du' = 0\,. \tag{A.77}$$

[Note that if the descent stops in form degree zero, $\deg(v) = 0$, then the proof remains valid with the sole modification $\nu' = 0$ in (A.74).] Reprising the same argument in relation with (A.75) and the last equation, we find that (A.77) can be replaced with

$$\gamma t' = 0\,, \tag{A.78}$$

where

$$t' = t - d\rho''\,, \tag{A.79}$$

and ρ'' comes from

$$u' = d\nu'' + \gamma\rho''\,, \qquad \gamma\nu'' = 0\,. \tag{A.80}$$

Performing exactly the same operations for the remaining equations from the descent, we finally infer that (A.61) is equivalent with

$$\gamma a' = 0\,, \tag{A.81}$$

where

$$a' = a - d\rho'''\,, \tag{A.82}$$

and ρ''' appears in

$$b' = d\nu''' + \gamma\rho'''\,, \qquad \gamma\nu''' = 0\,. \tag{A.83}$$

The corollary is now demonstrated once we perform the identification

$$\nu = -\rho'''\,, \tag{A.84}$$

between (A.82) and (A.63). Meanwhile, it is worth noticing that $b' = b - dg$, with γg nonvanishing in general, so from (A.83) we can also state that

$$b = \gamma\rho''' + df\,, \qquad f = \nu''' + g\,, \tag{A.85}$$

with $\gamma f \neq 0$ in general. $\square$

B. Some results on the (invariant) characteristic cohomology

We have argued in section 4.1 that the characteristic cohomology for the model under study is trivial in antighost numbers strictly greater than five, $H_k(\delta|d) = 0$ for all $k > 5$. It appears the natural question if this result is still valid in the space of invariant polynomials, or, in other words, at the level of the invariant characteristic cohomology $H^{\mathrm{inv}}(\delta|d)$. The answer is affirmative and is proved below, in Theorem 3. Actually, we prove that if α_k is trivial in $H_k(\delta|d)$, then it can be taken to be trivial also in $H_k^{\mathrm{inv}}(\delta|d)$. We consider only the case $k \geq 5$ since our main scope is to argue the triviality of $H^{\mathrm{inv}}(\delta|d)$ in antighost number strictly greater than five. To this end, we firstly need the following lemma.

Lemma 1. *Let α be a δ-exact invariant polynomial*

$$\alpha = \delta\beta\,. \tag{B.1}$$

Then, β can also be taken to be an invariant polynomial.

Proof. Let v be a function of $[\chi^*_\Delta]$, $[\varphi]$, $[A_\mu]$, $[H_\mu]$, $[B_{\mu\nu}]$, $[\phi_{\mu\nu}]$, and $[K_{\mu\nu\rho}]$. The dependence of v on $[\varphi]$, $[A_\mu]$, $[H_\mu]$, $[B_{\mu\nu}]$, $[\phi_{\mu\nu}]$, and $[K_{\mu\nu\rho}]$ can be reorganized as a dependence on $[F_A]$ and on $\bar{A}_\mu = \{A_\mu, \partial A_\mu, \ldots\}$, $\bar{H}_\mu = \{H_\mu, \partial H_\mu, \ldots\}$, $\bar{B}_{\mu\nu} = \{B_{\mu\nu}, \partial B_{\mu\nu}, \ldots\}$, $\bar{\phi}_{\mu\nu} = \{\phi_{\mu\nu}, \partial\phi_{\mu\nu}, \ldots\}$, $\bar{K}_{\mu\nu\rho} = \{K_{\mu\nu\rho}, \partial K_{\mu\nu\rho}, \ldots\}$, where, $\bar{A}_\mu$, $\bar{H}_\mu$, $\bar{B}_{\mu\nu}$, $\bar{\phi}_{\mu\nu}$, $\bar{K}_{\mu\nu\rho}$ are not γ-invariant. If v is γ-invariant, then it does not involve, $\bar{A}_\mu$, $\bar{H}_\mu$, $\bar{B}_{\mu\nu}$, $\bar{\phi}_{\mu\nu}$, $\bar{K}_{\mu\nu\rho}$, i.e., $v = v|_{\bar{A}_\mu=0,\bar{H}_\mu=0,\bar{B}_{\mu\nu}=0,\bar{\phi}_{\mu\nu}=0,\bar{K}_{\mu\nu\rho}=0}$, so we have by hypothesis that

$$\alpha = \alpha|_{\bar{A}_\mu=0,\bar{H}_\mu=0,\bar{B}_{\mu\nu}=0,\bar{\phi}_{\mu\nu}=0,\bar{K}_{\mu\nu\rho}=0} \,. \tag{B.2}$$

On the other hand, β depends in general on $[\chi^*_\Delta]$, $[F_A]$ and, $\bar{A}_\mu$, $\bar{H}_\mu$, $\bar{B}_{\mu\nu}$, $\bar{\phi}_{\mu\nu}$, $\bar{K}_{\mu\nu\rho}$. Making $\bar{A}_\mu = 0$, $\bar{H}_\mu = 0$, $\bar{B}_{\mu\nu} = 0$, $\bar{\phi}_{\mu\nu} = 0$, $\bar{K}_{\mu\nu\rho} = 0$ in (B.1), using (B.2) and taking into account the fact that δ commutes with the operation of setting the 'fields' $\bar{A}_\mu$, $\bar{H}_\mu$, $\bar{B}_{\mu\nu}$, $\bar{\phi}_{\mu\nu}$, and $\bar{K}_{\mu\nu\rho}$ equal to zero, we find that

$$\alpha = \delta\left(\beta|_{\bar{A}_\mu=0,\bar{H}_\mu=0,\bar{B}_{\mu\nu}=0,\bar{\phi}_{\mu\nu}=0,\bar{K}_{\mu\nu\rho}=0}\right), \tag{B.3}$$

with $\beta|_{\bar{A}_\mu=0,\bar{H}_\mu=0,\bar{B}_{\mu\nu}=0,\bar{\phi}_{\mu\nu}=0,\bar{K}_{\mu\nu\rho}=0}$ invariant. This proves the lemma. □

Now, we have the necessary tools for proving the next theorem.

Theorem 3. *Let α^p_k be an invariant polynomial with* $\deg\left(\alpha^p_k\right) = p$ *and* $agh\left(\alpha^p_k\right) = k \geq 5$, *which is δ-exact modulo d*

$$\alpha^p_k = \delta\lambda^p_{k+1} + d\lambda^{p-1}_k\,, \qquad k \geq 5\,. \tag{B.4}$$

Then, we can choose λ^p_{k+1} and λ^{p-1}_k to be invariant polynomials.

Proof. Initially, by successively acting with d and δ on (B.4) we obtain a tower of equations of the same type. Indeed, acting with d on (B.4) we find that $d\alpha^p_k = -\delta\left(d\lambda^p_{k+1}\right)$. On the other hand, as $d\alpha^p_k$ is invariant, by means of Lemma 1 we obtain that $d\alpha^p_k = -\delta\alpha^{p+1}_{k+1}$, with α^{p+1}_{k+1} invariant. The last two relations imply that $\delta\left(\alpha^{p+1}_{k+1} - d\lambda^p_{k+1}\right) = 0$. As δ is acyclic at strictly positive antighost numbers, the last relation implies that

$$\alpha^{p+1}_{k+1} = \delta\lambda^{p+1}_{k+2} + d\lambda^p_{k+1}\,. \tag{B.5}$$

Starting now with (B.5) and reprising the same operations like those performed between the formulas (B.4) and (B.5), we obtain a descent that stops in form degree 5 with the equation $\alpha^5_{k+5-p} = \delta\lambda^5_{k+5-p+1} + d\lambda^4_{k+5-p}$. Now, we act with δ on (B.4) and deduce that $\delta\alpha^p_k = -d\delta\lambda^{p-1}_k$. As $\delta\alpha^p_k$ is invariant, in the case $k > 1$, due to the Theorem 1, we obtain that $\delta\alpha^p_k = -d\alpha^{p-1}_{k-1}$, where α^{p-1}_{k-1} is invariant. Using the last two relations we get that $d\left(\alpha^{p-1}_{k-1} - \delta\lambda^{p-1}_k\right) = 0$, such that it follows that

$$\alpha^{p-1}_{k-1} = \delta\lambda^{p-1}_k + d\lambda^{p-2}_{k-1}\,. \tag{B.6}$$

If $k = 5$ in (B.4), we cannot go down since by assumption $k \geq 5$, and so the bottom of the tower is (B.4) for $k = 5$. Starting from (B.6) and reprising the same procedure we reach a descent that ends at either form degree zero or antighost number five, hence the last equation respectively takes the form

$$\alpha^{0}_{k-p} = \delta\lambda^{0}_{k-p+1}\,, \tag{B.7}$$

for $k - p \geq 5$ or

$$\alpha^{p-k+5}_{5} = \delta\lambda^{p-k+5}_{6} + d\lambda^{p-k+4}_{5}\,, \tag{B.8}$$

for $k - p < 5$. In consequence, the procedure described in the above leads to the chain

$$\begin{aligned}
\alpha^{5}_{k+5-p} &= \delta\lambda^{5}_{k+6-p} + d\lambda^{4}_{k+5-p}\,,\\
&\ \vdots\\
\alpha^{p+1}_{k+1} &= \delta\lambda^{p+1}_{k+2} + d\lambda^{p}_{k+1}\,,\\
\alpha^{p}_{k} &= \delta\lambda^{p}_{k+1} + d\lambda^{p-1}_{k}\,,\\
\alpha^{p-1}_{k-1} &= \delta\lambda^{p-1}_{k} + d\lambda^{p-2}_{k-1}\,,\\
&\ \vdots\\
\alpha^{0}_{k-p} &= \delta\lambda^{0}_{k-p+1} \qquad \text{or} \qquad \alpha^{p-k+5}_{5} = \delta\lambda^{p-k+5}_{6} + d\lambda^{p-k+4}_{5}\,.
\end{aligned} \tag{B.9}$$

All the α's in the descent (B.9) are invariant.

Now, we show that if one of the λ's in (B.9) is invariant, then all the other λ's can be taken to be also invariant. Indeed, let λ^{A-1}_{B} be invariant. It is involved in two of the equations from (B.9), namely

$$\alpha^{A}_{B} = \delta\lambda^{A}_{B+1} + d\lambda^{A-1}_{B}\,, \tag{B.10}$$

$$\alpha^{A-1}_{B-1} = \delta\lambda^{A-1}_{B} + d\lambda^{A-2}_{B-1}\,. \tag{B.11}$$

The relation (B.10) yields that $\alpha^{A}_{B} - d\lambda^{A-1}_{B}$ is invariant. Then, in agreement with Lemma 1 the object λ^{A}_{B+1} can be chosen to be invariant. Using (B.11), we have that $\alpha^{A-1}_{B-1} - \delta\lambda^{A-1}_{B}$ is invariant, such that Theorem 1 ensures that λ^{A-2}_{B-1} is also invariant. On the other hand, λ^{A}_{B+1} and λ^{A-2}_{B-1} are involved in other two sets of equations from the descent. (For instance, the former element appears in the equations $\alpha^{A+1}_{B+1} = \delta\lambda^{A+1}_{B+2} + d\lambda^{A}_{B+1}$ and $\alpha^{A+2}_{B+2} = \delta\lambda^{A+2}_{B+3} + d\lambda^{A+1}_{B+2}$.) Going on in the same fashion, we find that all the λ's are invariant. In the case where λ^{A-1}_{B} appears at the top or at the bottom of the descent, we act in a similar way, but only with respect to a single equation. The above considerations emphasize that it is enough to verify the theorem in form degree 5 and for all the values $k \geq 5$ of the antighost number.

If $k \geq 10$ (and hence $k - p \geq 5$), the last equation from the descent (B.9) for $p = 5$ reads as

$$\alpha^{0}_{k-5} = \delta\lambda^{0}_{k-4}\,. \tag{B.12}$$

Using Lemma 1, it results that λ^{0}_{k-4} can be taken to be invariant, such that the above arguments lead to the conclusion that all the λ's from the descent can be chosen invariant. As a consequence, in the first equation from the descent in this situation, namely, $\alpha^{5}_{k} =$

$\delta\lambda^5_{k+1} + d\lambda^4_k$, we have that both λ^5_{k+1} and λ^4_k are invariant. Therefore, the theorem is true in form degree 5 and in all antighost numbers $k \geq 10$, so it remains to be proved that it holds in form degree 5 and in all antighost numbers $5 \leq k < 10$. This is done below.

In the sequel we consider the case $p = 5$ and $5 \leq k < 10$. The top equation from (B.9), written in dual notations, takes the form

$$\alpha_k = \delta\lambda_{k+1} + \partial_\mu \lambda^\mu_k \,, \qquad 5 \leq k < 10 \,. \tag{B.13}$$

On the other hand, we can express α_k in terms of its E.L. derivatives by means of the homotopy formula

$$\alpha_k = \int_0^1 d\tau \left(\frac{\delta^R \alpha_k}{\delta \chi^*_\Delta} (\tau)\, \chi^*_\Delta + \frac{\delta \alpha_k}{\delta \Phi^{\alpha_0}} (\tau)\, \Phi^{\alpha_0} \right) + \partial_\mu j^\mu_k \,, \tag{B.14}$$

where $\frac{\delta^R \alpha_k}{\delta \chi^*_\Delta}(\tau) = \frac{\delta^R \alpha_k}{\delta \chi^*_\Delta}(\tau [F_A], \tau [\chi^*_\Delta])$ and similarly for the other terms. For further convenience, we denote the E.L. derivatives of λ_{k+1} by

$$\frac{\delta^R \lambda_{k+1}}{\delta C^*_{\mu\nu\rho\lambda\sigma}} = G^{\mu\nu\rho\lambda\sigma}_{k-4} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta C^*_{\mu\nu\rho\lambda}} = G^{\mu\nu\rho\lambda}_{k-3} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta C^*_{\mu\nu\rho}} = G^{\mu\nu\rho}_{k-2} \,, \tag{B.15}$$

$$\frac{\delta^R \lambda_{k+1}}{\delta C^*_{\mu\nu}} = G^{\mu\nu}_{k-1} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta H^*_\mu} = G^\mu_k \,, \quad \frac{\delta \lambda_{k+1}}{\delta H_\mu} = \bar{G}^\mu_{k+1} \,, \tag{B.16}$$

$$\frac{\delta \lambda_{k+1}}{\delta \varphi^*} = M_k \,, \quad \frac{\delta \lambda_{k+1}}{\delta \varphi} = \bar{M}_{k+1} \,, \tag{B.17}$$

$$\frac{\delta^R \lambda_{k+1}}{\delta \eta^*} = N_{k-1} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta A^*_\mu} = N^\mu_k \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta A_\mu} = \bar{N}^\mu_{k+1} \,, \tag{B.18}$$

$$\frac{\delta^R \lambda_{k+1}}{\delta \tilde{\eta}^*} = Q_{k-3} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \tilde{\eta}^*_\mu} = Q^\mu_{k-2} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \tilde{\eta}^*_{\mu\nu}} = Q^{\mu\nu}_{k-1} \,, \tag{B.19}$$

$$\frac{\delta^R \lambda_{k+1}}{\delta \tilde{B}^*_{\mu\nu\rho}} = Q^{\mu\nu\rho}_k \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \tilde{B}_{\mu\nu\rho}} = \bar{Q}^{\mu\nu\rho}_{k+1} \,, \tag{B.20}$$

$$\frac{\delta^R \lambda_{k+1}}{\delta C^*} = L_{k-2} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta C^*_\mu} = L^\mu_{k-1} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \phi^*_{\mu\nu}} = L^{\mu\nu}_k \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \phi_{\mu\nu}} = \bar{L}^{\mu\nu}_{k+1} \,, \tag{B.21}$$

$$\frac{\delta^R \lambda_{k+1}}{\delta \overset{\sim}{\mathcal{G}}{}^*} = P_{k-2} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \overset{\sim}{\mathcal{G}}{}^*_\mu} = P^\mu_{k-1} \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \tilde{K}^*_{\mu\nu}} = P^{\mu\nu}_k \,, \quad \frac{\delta^R \lambda_{k+1}}{\delta \tilde{K}_{\mu\nu}} = \bar{P}^{\mu\nu}_{k+1} \,. \tag{B.22}$$

Using (B.13), as well as the homotopy formula for λ_{k+1}, together with the notations (B.15)–(B.17), we determine the relationship between the E.L. derivatives of α_k and those of λ_{k+1} in the (H^μ, φ)-field/antifield sector like

$$\frac{\delta^R \alpha_k}{\delta C^*_{\mu\nu\rho\lambda\sigma}} = -\delta G^{\mu\nu\rho\lambda\sigma}_{k-4} \,, \qquad \frac{\delta^R \alpha_k}{\delta C^*_{\mu\nu\rho\lambda}} = \delta G^{\mu\nu\rho\lambda}_{k-3} + 5\partial_\sigma G^{\mu\nu\rho\lambda\sigma}_{k-4} \,, \tag{B.23}$$

$$\frac{\delta^R \alpha_k}{\delta C^*_{\mu\nu\rho}} = -\delta G^{\mu\nu\rho}_{k-2} + 4\partial_\lambda G^{\mu\nu\rho\lambda}_{k-3} \,, \qquad \frac{\delta^R \alpha_k}{\delta C^*_{\mu\nu}} = \delta G^{\mu\nu}_{k-1} + 3\partial_\rho G^{\mu\nu\rho}_{k-2} \,, \tag{B.24}$$

$$\frac{\delta^R \alpha_k}{\delta H^*_\mu} = -\delta G^\mu_k + 2\partial_\nu G^{\mu\nu}_{k-1} \,, \qquad \frac{\delta \alpha_k}{\delta H_\mu} = \delta \bar{G}^\mu_{k+1} - \partial^\mu M_k \,, \tag{B.25}$$

$$\frac{\delta\alpha_k}{\delta\varphi^*} = -\delta M_k\,, \qquad \frac{\delta\alpha_k}{\delta\varphi} = \delta\bar{M}_{k+1}\,. \tag{B.26}$$

Due to Lemma 1 ($k > 0$), the equations (B.26) allow us to state that

$$\frac{\delta\alpha_k}{\delta\varphi^*} = -\delta M'_k\,, \qquad \frac{\delta\alpha_k}{\delta\varphi} = \delta\bar{M}'_{k+1}\,, \tag{B.27}$$

with both M'_k and $\bar{M}'_{k+1}$ invariant polynomials. Applying a similar reasoning in connection with the descent (B.23)–(B.26) from bottom to top, we obtain that

$$\frac{\delta^R\alpha_k}{\delta C^*_{\mu\nu\rho\lambda\sigma}} = -\delta G'^{\mu\nu\rho\lambda\sigma}_{k-4}\,, \qquad \frac{\delta^R\alpha_k}{\delta C^*_{\mu\nu\rho\lambda}} = \delta G'^{\mu\nu\rho\lambda}_{k-3} + 5\partial_\sigma G'^{\mu\nu\rho\lambda\sigma}_{k-4}\,, \tag{B.28}$$

$$\frac{\delta^R\alpha_k}{\delta C^*_{\mu\nu\rho}} = -\delta G'^{\mu\nu\rho}_{k-2} + 4\partial_\lambda G'^{\mu\nu\rho\lambda}_{k-3}\,, \qquad \frac{\delta^R\alpha_k}{\delta C^*_{\mu\nu}} = \delta G'^{\mu\nu}_{k-1} + 3\partial_\rho G'^{\mu\nu\rho}_{k-2}\,, \tag{B.29}$$

$$\frac{\delta^R\alpha_k}{\delta H^*_\mu} = -\delta G'^{\mu}_{k} + 2\partial_\nu G'^{\mu\nu}_{k-1}\,, \qquad \frac{\delta\alpha_k}{\delta H_\mu} = \delta\bar{G}'^{\mu}_{k+1} - \partial^\mu M'_k\,, \tag{B.30}$$

where all the 'prime' quantities are invariant polynomials. On the other hand, since α_k is an invariant polynomial that depends on H_μ just through $\partial_\mu H^\mu$ and its spacetime derivatives, we get that

$$\frac{\delta\alpha_k}{\delta H_\mu} = \partial^\mu\Delta_k\,. \tag{B.31}$$

Using now the last equation from (B.30) together with (B.31), we arrive at

$$\delta\bar{G}'^{\mu}_{k+1} = \partial^\mu\left(M'_k + \Delta_k\right), \tag{B.32}$$

which indicates that $\bar{G}'^{\mu}_{k+1}$ belongs to $H^1_{k+1}(\delta|d)$. As $H^1_{k+1}(\delta|d) \simeq H^2_{k+2}(\delta|d) \simeq H^3_{k+3}(\delta|d) \simeq H^4_{k+4}(\delta|d) \simeq H^5_{k+5}(\delta|d)$ and $H^5_{k+5}(\delta|d) \simeq 0$, the equation (B.32) further implies

$$\bar{G}'^{\mu}_{k+1} = \delta\bar{G}''^{\mu}_{k+2} + \partial^\mu M''_{k+1}\,. \tag{B.33}$$

We will prove the theorem for $5 \leq k < 10$ by induction. More precisely, we will assume that the theorem holds in antighost numbers $(k+2)$, $(k+3)$, $(k+4)$, $(k+5)$, and in form degree 5, and will prove that it is also valid in antighost number k and in form degree 5. In agreement with this inductive hypothesis [more precisely, that the theorem is satisfied in antighost number $(k+5)$ and in form degree 5], we can take both $\bar{G}''^{\mu}_{k+2}$ and M''_{k+1} in (B.33) to be invariant polynomials.

From (B.13), the homotopy formula for λ_{k+1} and the definitions (B.18)–(B.20) we deduce the relationship between the E.L. derivatives of α_k and those of λ_{k+1} in the $(B^{\mu\nu}, A^\mu)$-field/antifield sector. It is more convenient to work, instead of the field $B^{\mu\nu}$ and of the antifields of the ghosts associated with its gauge invariance and the accompanying reducibility relations, with their Hodge duals, denoted by tilde variables. In terms of these new fields we have that

$$\frac{\delta^R\alpha_k}{\delta\eta^*} = \delta N_{k-1}\,, \qquad \frac{\delta^R\alpha_k}{\delta A^*_\mu} = -\delta N^\mu_k + \partial^\mu N_{k-1}\,, \tag{B.34}$$

$$\frac{\delta^R \alpha_k}{\delta A_\mu} = \delta \bar{N}^\mu_{k+1} - \frac{1}{8}\sigma^{\mu\alpha}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\partial^{[\beta}Q^{\gamma\delta\varepsilon]}_k \,, \tag{B.35}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{\eta}^*} = \delta Q_{k-3} \,, \qquad \frac{\delta^R \alpha_k}{\delta \tilde{\eta}^*_\mu} = -\delta Q^\mu_{k-2} - \partial^\mu Q_{k-3} \,, \tag{B.36}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{\eta}^*_{\mu\nu}} = \delta Q^{\mu\nu}_{k-1} - \frac{1}{2}\partial^{[\mu}Q^{\nu]}_{k-2} \,, \qquad \frac{\delta^R \alpha_k}{\delta \tilde{B}^*_{\mu\nu\rho}} = -\delta Q^{\mu\nu\rho}_k - \frac{1}{3}\partial^{[\mu}Q^{\nu\rho]}_{k-1} \,, \tag{B.37}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{B}_{\mu\nu\rho}} = \delta \bar{Q}^{\mu\nu\rho}_{k+1} + \frac{1}{12}\varepsilon^{\mu\nu\rho\lambda\sigma}\sigma_{\lambda\alpha}\sigma_{\sigma\beta}\partial^{[\alpha}N^{\beta]}_k \,. \tag{B.38}$$

Due to the Lemma 1 and on account of the fact that the E.L. derivatives of invariant polynomials are also invariant, the first equation in (B.34) and the former relation from (B.36) can be written like

$$\frac{\delta^R \alpha_k}{\delta \eta^*} = \delta N'_{k-1} \,, \qquad \frac{\delta^R \alpha_k}{\delta \tilde{\eta}^*} = \delta Q'_{k-3} \,, \tag{B.39}$$

where both N'_{k-1} and Q'_{k-3} are invariant. Reprising the same arguments for the remaining equations in (B.34)–(B.38), we infer that

$$\frac{\delta^R \alpha_k}{\delta A^*_\mu} = -\delta N'^\mu_k + \partial^\mu N'_{k-1} \,, \tag{B.40}$$

$$\frac{\delta^R \alpha_k}{\delta A_\mu} = \delta \bar{N}'^\mu_{k+1} - \frac{1}{8}\sigma^{\mu\alpha}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\partial^{[\beta}Q'^{\gamma\delta\varepsilon]}_k \,, \tag{B.41}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{\eta}^*_\mu} = -\delta Q'^\mu_{k-2} - \partial^\mu Q'_{k-3} \,, \tag{B.42}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{\eta}^*_{\mu\nu}} = \delta Q'^{\mu\nu}_{k-1} - \frac{1}{2}\partial^{[\mu}Q'^{\nu]}_{k-2} \,, \qquad \frac{\delta^R \alpha_k}{\delta \tilde{B}^*_{\mu\nu\rho}} = -\delta Q'^{\mu\nu\rho}_k - \frac{1}{3}\partial^{[\mu}Q'^{\nu\rho]}_{k-1} \,, \tag{B.43}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{B}_{\mu\nu\rho}} = \delta \bar{Q}'^{\mu\nu\rho}_{k+1} + \frac{1}{12}\varepsilon^{\mu\nu\rho\lambda\sigma}\sigma_{\lambda\alpha}\sigma_{\sigma\beta}\partial^{[\alpha}N'^{\beta]}_k \,, \tag{B.44}$$

where all the prime quantities are invariant. Let us analyze the relations (B.41) and (B.44). Since the invariant quantity α_k depends on A_μ through the combination $\partial_{[\alpha}A_{\beta]}$ and on $\tilde{B}_{\mu\nu\rho}$ via the expression $\partial_{[\alpha}\tilde{B}_{\beta\gamma\delta]}$, it follows that there exist some elements $\Delta^{\mu\nu}_k$ and $\Delta^{\mu\nu\rho\lambda}_k$, completely antisymmetric in their Lorentz indices, such that

$$\frac{\delta^R \alpha_k}{\delta A_\mu} = \partial_\nu \Delta^{\mu\nu}_k \,, \tag{B.45}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{B}_{\mu\nu\rho}} = \partial_\lambda \Delta^{\mu\nu\rho\lambda}_k \,. \tag{B.46}$$

Inserting (B.45) in (B.41) and (B.46) respectively in (B.44), we obtain the relations

$$\delta \bar{N}'^\mu_{k+1} = \partial_\nu \left(\Delta^{\mu\nu}_k + 3\tilde{Q}'^{\mu\nu}_k\right) , \tag{B.47}$$

$$\delta \bar{Q}'^{\mu\nu\rho}_{k+1} = \partial_\lambda \left(\Delta^{\mu\nu\rho\lambda}_k - \frac{1}{6}\tilde{N}'^{\mu\nu\rho\lambda}_k\right) , \tag{B.48}$$

where $\tilde{Q}_k^{\prime\mu\nu}$ is the Hodge dual of $Q_k^{\prime\mu\nu\rho}$, while $\tilde{N}_k^{\prime\mu\nu\rho\lambda}$ represents the Hodge dual of $N_k^{\prime\mu}$. In dual language, the equation (B.47) shows that $\bar{N}_{k+1}^{\prime\mu}$ belongs to $H_{k+1}^4(\delta|d)$. But $H_{k+1}^4(\delta|d) \simeq H_{k+2}^5(\delta|d)$ and $H_{k+2}^5(\delta|d) \simeq 0$, so the inductive hypothesis [in this case the validity of the theorem in antighost number $(k+2)$ and in form degree 5] allows us to conclude that there exist some invariant polynomials $\bar{N}_{k+2}^{\prime\prime\mu}$ and $\tilde{Q}_{k+1}^{\prime\prime\mu\nu}$, in terms of which we have that

$$\bar{N}_{k+1}^{\prime\mu} = \delta\bar{N}_{k+2}^{\prime\prime\mu} + \partial_\nu\tilde{Q}_{k+1}^{\prime\prime\mu\nu} . \tag{B.49}$$

In the same dual language, from (B.48) we read that $\bar{Q}_{k+1}^{\prime\mu\nu\rho} \in H_{k+1}^2(\delta|d) \simeq H_{k+2}^3(\delta|d) \simeq H_{k+3}^4(\delta|d) \simeq H_{k+4}^5(\delta|d) \simeq 0$. Using again the inductive hypothesis [here, that the theorem holds in antighost number $(k+4)$ and in form degree 5], we then get the existence of some invariant polynomials $\bar{Q}_{k+2}^{\prime\prime\mu\nu\rho}$ and $\tilde{N}_{k+1}^{\prime\prime\mu\nu\rho\lambda}$ with the property

$$\bar{Q}_{k+1}^{\prime\mu\nu\rho} = \delta\bar{Q}_{k+2}^{\prime\prime\mu\nu\rho} + \partial_\lambda\tilde{N}_{k+1}^{\prime\prime\mu\nu\rho\lambda} . \tag{B.50}$$

Now, we invoke once more the relation (B.13) and the homotopy formula for λ_{k+1}, which, combined with the definitions (B.21)–(B.22), provides the relationship between the E.L. derivatives of α_k and of λ_{k+1} in the $(K^{\mu\nu\rho}, \phi^{\mu\nu})$-field/antifield sector. Instead of $K^{\mu\nu\rho}$ and of the antifields corresponding to the ghosts associated with the gauge invariance of this field, we work with their Hodge duals, and deduce

$$\frac{\delta^R\alpha_k}{\delta C^*} = -\delta L_{k-2} , \qquad \frac{\delta^R\alpha_k}{\delta C_\mu^*} = \delta L_{k-1}^\mu - \partial^\mu L_{k-2} , \tag{B.51}$$

$$\frac{\delta^R\alpha_k}{\delta\phi_{\mu\nu}^*} = -\delta L_k^{\mu\nu} + \partial^{[\mu}L_{k-1}^{\nu]} , \qquad \frac{\delta^R\alpha_k}{\delta\phi_{\mu\nu}} = \delta\bar{L}_{k+1}^{\mu\nu} + \frac{1}{3}\partial_\rho\tilde{P}_k^{\mu\nu\rho} , \tag{B.52}$$

$$\frac{\delta^R\alpha_k}{\delta \overset{\sim}{\mathcal{G}}^*} = -\delta P_{k-2} , \qquad \frac{\delta^R\alpha_k}{\delta \overset{\sim}{\mathcal{G}}_\mu^*} = \delta P_{k-1}^\mu + \partial^\mu P_{k-2} , \tag{B.53}$$

$$\frac{\delta^R\alpha_k}{\delta\tilde{K}_{\mu\nu}^*} = -\delta P_k^{\mu\nu} + \frac{1}{2}\partial^{[\mu}P_{k-1}^{\nu]} , \qquad \frac{\delta^R\alpha_k}{\delta\tilde{K}_{\mu\nu}} = \delta\bar{P}_{k+1}^{\mu\nu} + \partial_\rho\tilde{L}_k^{\mu\nu\rho} , \tag{B.54}$$

where $\tilde{P}_k^{\mu\nu\rho}$ and $\tilde{L}_k^{\mu\nu\rho}$ are dual to $P_k^{\mu\nu}$ and respectively to $L_k^{\mu\nu}$. The Lemma 1 and the fact that the E.L. derivatives of any invariant polynomial is also invariant allow us to write the former equations in (B.51) and respectively in (B.53) like

$$\frac{\delta^R\alpha_k}{\delta C^*} = -\delta L_{k-2}^{\prime} , \qquad \frac{\delta^R\alpha_k}{\delta \overset{\sim}{\mathcal{G}}^*} = -\delta P_{k-2}^{\prime} , \tag{B.55}$$

with both $L_{k-2}^{\prime}$ and $P_{k-2}^{\prime}$ invariant. The same reasoning, extended to the remaining equations in (B.51)–(B.54), produces

$$\frac{\delta^R\alpha_k}{\delta C_\mu^*} = \delta L_{k-1}^{\prime\mu} - \partial^\mu L_{k-2}^{\prime} , \qquad \frac{\delta^R\alpha_k}{\delta\phi_{\mu\nu}^*} = -\delta L_k^{\prime\mu\nu} + \partial^{[\mu}L_{k-1}^{\prime\nu]} , \tag{B.56}$$

$$\frac{\delta^R\alpha_k}{\delta\phi_{\mu\nu}} = \delta\bar{L}_{k+1}^{\prime\mu\nu} + \frac{1}{3}\partial_\rho\tilde{P}_k^{\prime\mu\nu\rho} , \tag{B.57}$$

$$\frac{\delta^R \alpha_k}{\delta \overset{\sim *}{\mathcal{G}}_\mu} = \delta P'^{\mu}_{k-1} + \partial^\mu P'_{k-2}\,, \qquad \frac{\delta^R \alpha_k}{\delta \tilde{K}^*_{\mu\nu}} = -\delta P'^{\mu\nu}_{k} + \frac{1}{2}\partial^{[\mu} P'^{\nu]}_{k-1}\,, \tag{B.58}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{K}_{\mu\nu}} = \delta \bar{P}'^{\mu\nu}_{k+1} + \partial_\rho \tilde{L}'^{\mu\nu\rho}_{k}\,, \tag{B.59}$$

where all the prime objects from (B.56)–(B.59) are invariant polynomials. Let us focus on the relations (B.57) and (B.59). Due to the fact that the invariant element α_k depends on $\phi_{\mu\nu}$ through the combination $\partial_{[\alpha}\phi_{\beta\gamma]}$ and on $\tilde{K}_{\mu\nu}$ via the expression $\partial_{[\alpha}\tilde{K}_{\beta\gamma]}$, it results that there exist some objects $\Omega^{\mu\nu\rho}_k$ and $\Gamma^{\mu\nu\rho}_k$, completely antisymmetric in their Lorentz indices, such that

$$\frac{\delta^R \alpha_k}{\delta \phi_{\mu\nu}} = \partial_\rho \Omega^{\mu\nu\rho}_k\,, \tag{B.60}$$

$$\frac{\delta^R \alpha_k}{\delta \tilde{K}_{\mu\nu}} = \partial_\rho \Gamma^{\mu\nu\rho}_k\,. \tag{B.61}$$

Putting together the equation (B.60) with (B.57) and respectively the relation (B.61) with (B.59), we find that

$$\delta \bar{L}'^{\mu\nu}_{k+1} = \partial_\rho \left(\Omega^{\mu\nu\rho}_k - \frac{1}{3}\tilde{P}'^{\mu\nu\rho}_k\right), \tag{B.62}$$

$$\delta \bar{P}'^{\mu\nu}_{k+1} = \partial_\rho \left(\Gamma^{\mu\nu\rho}_k - \tilde{L}'^{\mu\nu\rho}_k\right), \tag{B.63}$$

which indicates that both the invariant polynomials $\bar{L}'^{\mu\nu}_{k+1}$ and $\bar{P}'^{\mu\nu}_{k+1}$ pertain to $H^3_{k+1}(\delta|d) \simeq H^4_{k+2}(\delta|d) \simeq H^5_{k+3}(\delta|d) \simeq 0$. The inductive hypothesis [more exactly, that the theorem is fulfilled in antighost number $(k+3)$ and in form degree 5] ensures now the existence of two sets of invariant polynomials, $\bar{L}''^{\mu\nu\rho}_{k+2}$, $\tilde{P}''^{\mu\nu\rho}_{k+1}$ and $\bar{P}''^{\mu\nu}_{k+2}$, $\tilde{L}''^{\mu\nu\rho}_{k}$, with the help of which we can write

$$\bar{L}'^{\mu\nu}_{k+1} = \delta \bar{L}''^{\mu\nu\rho}_{k+2} + \partial_\rho \tilde{P}''^{\mu\nu\rho}_{k+1}\,, \tag{B.64}$$

$$\bar{P}'^{\mu\nu}_{k+1} = \delta \bar{P}''^{\mu\nu}_{k+2} + \partial_\rho \tilde{L}''^{\mu\nu\rho}_{k}\,. \tag{B.65}$$

Introducing in (B.14) the E.L. derivatives of the invariant polynomial α_k, expressed via the relations (B.27)–(B.30), (B.39)–(B.44), and (B.55)–(B.59), we finally determine that

$$\begin{aligned}
\alpha_k = \delta\Bigg[\int_0^1 d\tau \Big(& G'^{\mu\nu\rho\lambda\sigma}_{k-4}(\tau)\, C^*_{\mu\nu\rho\lambda\sigma} + G'^{\mu\nu\rho\lambda}_{k-3}(\tau)\, C^*_{\mu\nu\rho\lambda} + G'^{\mu\nu\rho}_{k-2}(\tau)\, C^*_{\mu\nu\rho} + \\
& + G'^{\mu\nu}_{k-1}(\tau)\, C^*_{\mu\nu} + G'^{\mu}_{k}(\tau)\, H^*_\mu + M'_k(\tau)\, \varphi^* + \bar{M}'_{k+1}(\tau)\, \varphi - \\
& - M''_{k+1}(\tau)\, \partial^\mu H_\mu + N'_{k-1}(\tau)\, \eta^* + Q'_{k-3}(\tau)\, \tilde{\eta}^* + N'^{\mu}_{k}(\tau)\, A^*_\mu + \\
& + Q'^{\mu}_{k-2}(\tau)\, \tilde{\eta}^*_\mu + Q'^{\mu\nu}_{k-1}(\tau)\, \tilde{\eta}^*_{\mu\nu} + \frac{1}{4}\tilde{N}''^{\mu\nu\rho\lambda}_{k+1}(\tau)\, \partial_{[\mu}\tilde{B}_{\nu\rho\lambda]} + \\
& + Q'^{\mu\nu\rho}_{k}(\tau)\, \tilde{B}^*_{\mu\nu\rho} + \frac{1}{2}\tilde{Q}''^{\mu\nu}_{k+1}(\tau)\, \partial_{[\mu}A_{\nu]} + L'^{\mu}_{k-1}(\tau)\, C^*_\mu + \\
& + P'_{k-2}(\tau)\, \overset{\sim *}{\mathcal{G}} + L'^{\mu\nu}_{k}(\tau)\, \phi^*_{\mu\nu} + P'^{\mu}_{k-1}(\tau)\, \overset{\sim *}{\mathcal{G}}_\mu + P'^{\mu\nu}_{k}(\tau)\, \tilde{K}^*_{\mu\nu} +
\end{aligned}$$

$$+ L'_{k-2}(\tau)\, C^{*} - \frac{1}{3}\tilde{P}''^{\mu\nu\rho}_{k+1}(\tau)\, \partial_{[\mu}\phi_{\nu\rho]} - \frac{1}{3}\tilde{L}''^{\mu\nu\rho}_{k}(\tau)\, \partial_{[\mu}\tilde{K}_{\nu\rho]}\Big)\Big] +$$

$$+\partial_\mu \psi^\mu_k \,. \tag{B.66}$$

We observe that all the terms from the integrand are invariant. In order to prove that the current ψ^μ_k can also be taken invariant, we switch (B.66) to the original form notation

$$\alpha^5_k = \delta\lambda^5_{k+1} + d\lambda^4_k \,, \tag{B.67}$$

(where λ^4_k is dual to ψ^μ_k). As α^5_k is by assumption invariant and we have shown that λ^5_{k+1} can be taken invariant, (B.67) becomes

$$\beta^5_k = d\lambda^4_k \,. \tag{B.68}$$

It states that the invariant polynomial $\beta^5_k = \alpha^5_k - \delta\lambda^5_{k+1}$, of form degree 5 and of strictly positive antighost number, is d-exact. Then, in agreement with the Theorem 1 in form degree 5 (see the paragraph following this theorem), we can take λ^4_k (or, which is the same, ψ^μ_k) to be invariant. In conclusion, the induction hypothesis in antighost numbers $(k+2)$, $(k+3)$, $(k+4)$, $(k+5)$, and form degree 5 leads to the same property for antighost number k and form degree 5, which proves the theorem for all $k \geq 5$ since we have shown that it holds for $k \geq 10$. □

The most important consequence of the last theorem is the validity of the result (4.15) on the triviality of $H^{\mathrm{inv}}(\delta|d)$ in antighost number strictly greater than five.

C. Local cohomology of s, $H(s|d)$

Now, we have all the necessary tools for the study of the local cohomology $H(s|d)$ in form degree 5. We will show that it is always possible to remove the components of antighost number strictly greater than five from any co-cycle of $H^g_5(s|d)$ in form degree five only by trivial redefinitions.

We consider a co-cycle from $H^g_5(s|d)$, $sa + db = 0$, with $\deg(a) = 5$, $\mathrm{gh}(a) = g$, $\deg(b) = 4$, $\mathrm{gh}(b) = g+1$. Trivial redefinitions of a and b mean the simultaneous transformations $a \to a + sc + de$ and $b \to b + df + se$. We expand a and b according to the antighost number and ask that a_0 is local, such that each expansion stops at some finite antighost number [39], $a = \sum_{k=0}^{I} a_k$, $b = \sum_{k=0}^{M} b_k$, $\mathrm{agh}(a_k) = k = \mathrm{agh}(b_k)$. Due to the splitting $s = \delta + \gamma$, the equation $sa + db = 0$ is equivalent to the tower of equations

$$\begin{aligned}
\delta a_1 + \gamma a_0 + db_0 &= 0\,, \\
&\vdots \\
\delta a_I + \gamma a_{I-1} + db_{I-1} &= 0\,, \\
&\vdots
\end{aligned}$$

The form of the last equation depends on the values of I and M, but we can assume, without loss of generality, that $M = I - 1$. Indeed, if $M > I - 1$, the last $(M - I)$

equations read as $db_k = 0$, $I < k \leq M$, which imply that $b_k = df_k$, $\deg(f_k) = 3$. We can thus absorb all the pieces $(df_k)_{I<k\leq M}$ in a trivial redefinition of b, such that the new "current" stops at antighost number I. Accordingly, the bottom equation becomes $\gamma a_I + db_I = 0$, so the Corollary 1 ensures that we can make a redefinition $a_I \to a_I - d\rho_I$ such that $\gamma(a_I - d\rho_I) = 0$. Meanwhile, the same corollary [see the formula (A.85)] leads to $b_I = dg_I + \gamma\rho_I$, where $\deg(\rho_I) = 4$, $\deg(g_I) = 3$, $\mathrm{agh}(\rho_I) = \mathrm{agh}(g_I) = I$, $\mathrm{gh}(\rho_I) = g$, $\mathrm{gh}(g_I) = g+1$. Then, it follows that we can make the trivial redefinitions $a \to a - d\rho_I$ and $b \to b - dg_I - s\rho_I$, such that the new "current" stops at antighost number $(I-1)$, while the last component of the co-cycle from $H_5^g(s|d)$ is γ-closed.

In consequence, we obtained the equation $sa + db = 0$, with

$$a = \sum_{k=0}^{I} a_k\,, \qquad b = \sum_{k=0}^{I-1} b_k\,, \tag{C.1}$$

where $\mathrm{agh}(a_k) = k$ for $0 < k < I$ and $\mathrm{agh}(b_k) = k$ for $0 < k < I-1$. All a_k are 5-forms of ghost number g and all b_k are 4-forms of ghost number $(g+1)$, with $\mathrm{pgh}(a_k) = g+k$ for $0 < k < I$ and $\mathrm{pgh}(b_k) = g+k+1$ for $0 < k < I-1$. The equation $sa + db = 0$ is now equivalent with the tower of equations [where some $(b_k)_{0\leq k\leq I-1}$ could vanish]

$$\delta a_1 + \gamma a_0 + db_0 = 0\,, \tag{C.2}$$

$$\vdots$$

$$\delta a_{k+1} + \gamma a_k + db_k = 0\,, \tag{C.3}$$

$$\vdots$$

$$\delta a_I + \gamma a_{I-1} + db_{I-1} = 0\,, \tag{C.4}$$

$$\gamma a_I = 0\,. \tag{C.5}$$

Next, we show that we can eliminate all the terms $(a_k)_{k>5}$ and $(b_k)_{k>4}$ from the expansions (C.1) by trivial redefinitions only.

We can thus assume, without loss of generality, that any co-cycle a from $H_5^g(s|d)$ can be taken to stop at a value $I > 5$ of the antighost number. The last equation from the system equivalent with $sa + db = 0$ takes the form (C.5), with $\mathrm{pgh}(a_I) = g + I = L$, so $a_I \in H^L(\gamma)$. In agreement with the general results on $H(\gamma)$ (see section A) it follows that

$$a_I = \sum_J \alpha_J e^J + \gamma \bar{a}_I\,, \tag{C.6}$$

where α_J are invariant polynomials satisfying the properties

$$\mathrm{agh}(\alpha_J) = I\,, \qquad \deg(\alpha_J) = 5\,, \tag{C.7}$$

and e^J are the elements of pure ghost number equal to L of a basis in the space of polynomials in the ghosts η, C, $\widetilde{\mathcal{G}}$, $\tilde{\eta}$, $\tilde{C}$. By acting with the operator γ from the left on (C.4) and taking into account its second-order nilpotency, as well as its anticommutation with the exterior spacetime differential $\gamma d + d\gamma = 0$, one obtains that $-d(\gamma b_{I-1}) = 0$. The triviality of the cohomology of the differential d in the space of local forms in form degree equal to 4 leads to

$$\gamma b_{I-1} + dc_{I-1} = 0\,. \tag{C.8}$$

By means of the Corollary 1, it follows (as $I > 5$ by assumption, so $I-1>0$) that we can make a trivial redefinition such that (C.8) is replaced with the equation

$$\gamma b_{I-1} = 0\,. \tag{C.9}$$

In agreement with (C.9), b_{I-1} belongs to $H^L(\gamma)$, so we can take

$$b_{I-1} = \sum_J \beta_J e^J + \gamma \bar{b}_{I-1}\,, \tag{C.10}$$

where β_J are invariant polynomials with

$$\mathrm{agh}\,(\beta_J) = I-1\,, \qquad \deg\,(\beta_J) = 4\,, \tag{C.11}$$

and e^J is the same notation like in (C.6). Inserting (C.6) and (C.10) in (C.4) one infers

$$\pm \sum_J \left(\delta\alpha_J + d\beta_J\right) e^J = -\gamma \left(a_{I-1} + \sum_J \beta_J \hat{e}^J - \delta\bar{a}_I - d\bar{b}_{I-1} \right), \tag{C.12}$$

where $\hat{e}^J$ has been previously defined via the relation (A.54). Since the left-hand side of (C.12) is a nontrivial object from $H^L(\gamma)$, the equation (C.12) implies

$$\delta\alpha_J = -d\beta_J \qquad \text{for all } J\,. \tag{C.13}$$

The relation (C.13) shows that the invariant polynomials α_J belong to the space $H_I^5(\delta|d)$. As $I > 5$ by assumption and $H_I^5(\delta|d) = 0$ for $I > 5$, it follows that all the invariant polynomials α_J are trivial

$$\alpha_J = \delta\lambda^5_{I+1,J} + d\lambda^4_{I,J}\,, \tag{C.14}$$

where $\lambda^5_{I+1,J}$ are 5-forms of antighost number $(I+1)$, while $\lambda^4_{I,J}$ are 4-forms with the antighost number equal to I. Theorem 3 then ensures that we can also take $\lambda^5_{I+1,J}$ and $\lambda^4_{I,J}$ to be invariant polynomials, and thus α_J are in fact trivial elements of $H_I^{\mathrm{inv}5}(\delta|d)$. Replacing (C.14) in (C.13) and using the relations $\delta^2 = 0$ and $\delta d + d\delta = 0$, we deduce that $d\left(-\delta\lambda^4_{I,J} + \beta_J\right) = 0$. Because both $\lambda^4_{I,J}$ and β_J are invariant polynomials with strictly positive values of the antighost number and with the form degree equal to 4, Theorem 1 yields $-\delta\lambda^4_{I,J} + \beta_J = d\lambda^3_{I-1,J}$, where $\lambda^3_{I-1,J}$ are also invariant polynomials with $\mathrm{agh}\left(\lambda^3_{I-1,J}\right) = I-1$ and $\deg\left(\lambda^3_{I-1,J}\right) = 3$, such that we can write

$$\beta_J = \delta\lambda^4_{I,J} + d\lambda^3_{I-1,J}\,. \tag{C.15}$$

Substituting (C.14) in (C.6), after some computation we get that a_I is expressed in the form

$$a_I = \sum_J \left(\delta\lambda^5_{I+1,J} + d\lambda^4_{I,J}\right) e^J$$

$$= s\left(\pm\sum_J \lambda^5_{I+1,J} e^J\right) + d\left(\pm\sum_J \lambda^4_{I,J} e^J\right) \mp \sum_J \left(\lambda^4_{I,J} de^J\right). \tag{C.16}$$

Due to the fact that $de^J = \gamma \hat{e}^J$ and $\gamma\lambda^4_{I,J} = 0$, we consequently have that

$$a_I = s\left(\pm\sum_J \lambda^5_{I+1,J} e^J\right) + d\left(\pm\sum_J \lambda^4_{I,J} e^J\right) \mp \gamma\left(\sum_J \lambda^4_{I,J} \hat{e}^J\right). \tag{C.17}$$

In a similar manner, with the help of the relation (C.15) inserted in (C.10), we arrive to

$$b_{I-1} = s\left(\pm\sum_J \lambda^4_{I,J} e^J\right) + d\left(\pm\sum_J \lambda^3_{I-1,J} e^J\right) \mp \gamma\left(\sum_J \lambda^3_{I-1,J} \hat{e}^J\right). \tag{C.18}$$

Now, if we simultaneously perform some trivial redefinitions of a_I and of the 'current' b_{I-1} like

$$a'_I = a_I - s\left(\pm\sum_J \lambda^5_{I+1,J} e^J\right) - d\left(\pm\sum_J \lambda^4_{I,J} e^J\right), \tag{C.19}$$

$$b'_{I-1} = b_{I-1} - s\left(\pm\sum_J \lambda^4_{I,J} e^J\right) - d\left(\pm\sum_J \lambda^3_{I-1,J} e^J\right), \tag{C.20}$$

and, meanwhile, fix $\bar{a}_I$ and $\bar{b}_{I-1}$ from (C.6) and respectively from (C.10) as

$$\bar{a}_I = \pm\sum_J \lambda^4_{I,J} \hat{e}^J, \tag{C.21}$$

$$\bar{b}_{I-1} = \pm\sum_J \lambda^3_{I-1,J} \hat{e}^J, \tag{C.22}$$

then both a'_I and b'_{I-1} become equal to zero. Reprising now exactly the same procedure like before, but for the antighost number equal to $(I-1)$, we find that we can take $a_{I-1} = 0 = b_{I-2}$ by trivial redefinitions only. The elimination procedure stops at $k = 5$ in the tower (C.2)–(C.5) since for $k = 5$ we cannot pass from (C.13) to (C.14) as $H_5(\delta|d) \neq 0$. In conclusion, we can replace a_k with $a_k = 0$ by trivial redefinitions for all $k > 5$ in the tower (C.2)–(C.5), such that the first-order deformation can always be taken to end at $I = 5$ (formula (4.25)), with a_5 from $H(\gamma)$, $\gamma a_5 = 0$. Furthermore, the above arguments show that a_5 can be assumed to involve only non-trivial elements from $H^{\text{inv}}(\delta|d)$.

D. Solution to the 'homogeneous' equation in antighost number zero

In the sequel we consider the consistent interactions that do not modify the original gauge transformations, which are solutions to the equation

$$\gamma\bar{a}_0 = \partial_\mu j_0^\mu, \tag{D.1}$$

where $\bar{a}_0$ has the ghost number and the pure ghost number equal to zero. The solutions to this 'homogeneous' equation come from $\bar{a}_1 = 0$, and hence they bring contributions only to the deformed lagrangian density at order one in the coupling constant. We maintain all the hypotheses introduced in the beginning of section 4 (smoothness, locality, etc.), including the condition on the maximum derivative order of $\bar{a}_0$ being equal to one. There are two main types of solutions to this equation.

The first type, to be denoted by $\bar{a}'_0$, corresponds to $j_0^\mu = 0$, and is given by gauge-invariant, non-integrated densities constructed out of the original fields and their spacetime derivatives, which, according to (4.9), are of the form

$$\bar{a}'_0 = \bar{a}'_0 \left([F_{\bar{A}}] \right) , \tag{D.2}$$

where $\bar{a}'_0$ may contain at most one derivative of the fields. The sole possibility that complies with all the hypotheses on the interactions mentioned before is

$$\bar{a}'_0 = \bar{M} \left(\varphi \right) + N \left(\varphi \right) \partial_\mu H^\mu , \tag{D.3}$$

where $\bar{M}(\varphi)$ and $N(\varphi)$ are smooth, arbitrary real functions depending only on the undifferentiated scalar field. The second term in the right-hand side of (D.3) is δ-exact

$$N \left(\varphi \right) \partial_\mu H^\mu = \delta \left(N \left(\varphi \right) \varphi^* \right) ,$$

and hence produces trivial interactions, that can be eliminated via field redefinitions. This is due to the isomorphism $H^k(s|d) \simeq H^k(\gamma|d, H_0(\delta))$ in all positive values of the ghost number and respectively of the pure ghost number [40], which at $k = 0$ allows one to state that any solution of the homogeneous equation (D.1) that is δ-exact modulo d is in fact a trivial co-cycle from $H^0(s|d)$. In conclusion, the only nontrivial solution to (D.1) for $j_0^\mu = 0$ is represented by

$$\bar{a}'_0 = \bar{M} \left(\varphi \right) . \tag{D.4}$$

Although this solution does not contribute to the deformed gauge transformations of the interacting model, however it is important since it is involved in the consistency of the first-order deformation, as we have seen in the subsection 4.3 [see formula (4.32)].

The second kind of solutions, to be denoted by $\bar{a}''_0$, is associated with $j_0^\mu \neq 0$ in (D.1), being understood that we discard the divergence-like quantities and work under the hypothesis on the maximum derivative order of the interacting lagrangian being equal to one. Then, $\bar{a}''_0$ can be decomposed like

$$\bar{a}_0^{\prime\prime(\mathrm{int})} = \omega_0 + \omega_1 \tag{D.5}$$

where $(\omega_i)_{i=0,1}$ contains i derivatives of the fields. Due to the different number of derivatives in the components ω_0 and ω_1, the equation (D.1) leads to two independent equations

$$\gamma\omega_0 = \partial_\mu m_0^\mu , \tag{D.6}$$
$$\gamma\omega_1 = \partial_\mu m_1^\mu . \tag{D.7}$$

Since ω_0 is derivative-free, we find that

$$\gamma\omega_0 = \frac{\partial\omega_0}{\partial H^\mu} \left(2\partial_\nu C^{\mu\nu} \right) + \frac{\partial\omega_0}{\partial A_\mu} \left(\partial_\mu \eta \right) + \frac{\partial\omega_0}{\partial B^{\mu\nu}} \left(-3\partial_\rho \eta^{\mu\nu\rho} \right) +$$

$$+\frac{\partial\omega_0}{\partial\phi_{\mu\nu}}\left(\partial_{[\mu}C_{\nu]}\right)+\frac{\partial\omega_0}{\partial K^{\mu\nu\rho}}\left(4\partial_\lambda\mathcal{G}^{\mu\nu\rho\lambda}\right). \tag{D.8}$$

The right-hand side of the last equation reduces to a full derivative if the following conditions are simultaneously satisfied

$$\partial_{[\nu}\left(\frac{\partial\omega_0}{\partial H^{\mu]}}\right)=0\,,\qquad \partial_\mu\left(\frac{\partial\omega_0}{\partial A_\mu}\right)=0\,,\qquad \partial_{[\mu}\left(\frac{\partial\omega_0}{\partial B^{\nu\rho]}}\right)=0\,, \tag{D.9}$$

$$\partial_\mu\left(\frac{\partial\omega_0}{\partial\phi_{\mu\nu}}\right)=0\,,\qquad \partial_{[\mu}\left(\frac{\partial\omega_0}{\partial K^{\nu\rho\lambda]}}\right)=0\,. \tag{D.10}$$

Since neither of the functions present in (D.9) and (D.10) on which the derivatives act contain spacetime derivatives, these equations possess the solutions

$$\frac{\partial\omega_0}{\partial H^\mu}=c_\mu\,,\qquad \frac{\partial\omega_0}{\partial A_\mu}=k^\mu\,,\qquad \frac{\partial\omega_0}{\partial B^{\mu\nu}}=c_{\mu\nu}\,, \tag{D.11}$$

$$\frac{\partial\omega_0}{\partial\phi_{\mu\nu}}=k^{\mu\nu}\,,\qquad \frac{\partial\omega_0}{\partial K^{\mu\nu\rho}}=c_{\mu\nu\rho}\,, \tag{D.12}$$

where c_μ, k^μ, $c_{\mu\nu}$, $k^{\mu\nu}$, and $c_{\mu\nu\rho}$ are real, non-derivative constants, the last three sets being completely antisymmetric. Since there are no such constants, they must vanish, and therefore we have that $\omega_0=\omega_0(\varphi)$, which is nothing but the most general solution to (D.1) for $j_0^\mu=0$, given in (D.4). In conclusion, there is no consistent solution to the equation (D.1) for $j_0^\mu\neq 0$ that contains no derivatives of the fields, so we can take, without loss of generality, $\omega_0=0$ in the expansion (D.5).

The last step is to determine the consistent solutions ω_1 to (D.7) for $m_1^\mu\neq 0$ with precisely one spacetime derivative. In view of this, the general representation of ω_1 is $\omega_1([\varphi],[H^\mu],[B^{\mu\nu}],[A_\mu],[\phi_{\mu\nu}],[K^{\mu\nu\rho}])$. However, one can always move the single derivative present in ω_1 to act on all the fields but φ, so we will work with

$$\omega_1\left(\varphi,[H^\mu],[B^{\mu\nu}],[A_\mu],[\phi_{\mu\nu}],[K^{\mu\nu\rho}]\right). \tag{D.13}$$

Taking into account the definitions of γ, we obtain that

$$\begin{aligned}\gamma\omega_1 = {} & \frac{\partial\omega_1}{\partial H^\mu}\left(2\partial_\nu C^{\mu\nu}\right)+\frac{\partial\omega_1}{\partial\left(\partial^\alpha H^\mu\right)}\left(2\partial_\nu\partial^\alpha C^{\mu\nu}\right)+\frac{\partial\omega_1}{\partial B^{\mu\nu}}\left(-3\partial_\rho\eta^{\mu\nu\rho}\right)+\\ & +\frac{\partial\omega_1}{\partial\left(\partial^\alpha B^{\mu\nu}\right)}\left(-3\partial_\rho\partial^\alpha\eta^{\mu\nu\rho}\right)+\frac{\partial\omega_1}{\partial A_\mu}\left(\partial_\mu\eta\right)+\frac{\partial\omega_1}{\partial\left(\partial^\alpha A_\mu\right)}\left(\partial_\mu\partial^\alpha\eta\right)+\\ & +\frac{\partial\omega_1}{\partial\phi_{\mu\nu}}\left(\partial_{[\mu}C_{\nu]}\right)+\frac{\partial\omega_1}{\partial\left(\partial^\alpha\phi_{\mu\nu}\right)}\left(\partial^\alpha\partial_{[\mu}C_{\nu]}\right)+\\ & +\frac{\partial\omega_1}{\partial K^{\mu\nu\rho}}\left(4\partial_\lambda\mathcal{G}^{\mu\nu\rho\lambda}\right)+\frac{\partial\omega_1}{\partial\left(\partial^\alpha K^{\mu\nu\rho}\right)}\left(4\partial_\lambda\partial^\alpha\mathcal{G}^{\mu\nu\rho\lambda}\right).\end{aligned} \tag{D.14}$$

By successively moving all the derivatives from the ghosts, we observe that the right-hand side of (D.14) reduces to a total divergence if

$$\partial_{[\mu}\left(\frac{\delta\omega_1}{\delta H^{\nu]}}\right)=0\,,\qquad \partial_\mu\left(\frac{\delta\omega_1}{\delta A_\mu}\right)=0\,,\qquad \partial_{[\mu}\left(\frac{\delta\omega_1}{\delta B^{\nu\rho]}}\right)=0\,, \tag{D.15}$$

$$\partial_\mu \left(\frac{\delta \omega_1}{\delta \phi_{\mu\nu}} \right) = 0 \,, \qquad \partial_{[\mu} \left(\frac{\delta \omega_1}{\delta K^{\nu\rho\lambda]}} \right) = 0 \,. \tag{D.16}$$

The general solutions to the equations (D.15) and (D.16) are

$$\frac{\delta \omega_1}{\delta H^\mu} = \partial_\mu D \,, \qquad \frac{\delta \omega_1}{\delta A_\mu} = \partial_\nu D^{\mu\nu} \,, \qquad \frac{\delta \omega_1}{\delta B^{\mu\nu}} = \partial_{[\mu} D_{\nu]} \,, \tag{D.17}$$

$$\frac{\delta \omega_1}{\delta \phi_{\mu\nu}} = \partial_\rho E^{\mu\nu\rho} \,, \qquad \frac{\delta \omega_1}{\delta K^{\mu\nu\rho}} = \partial_{[\mu} E_{\nu\rho]} \,, \tag{D.18}$$

where in our case the functions D, D_μ, $D^{\mu\nu}$, $E^{\mu\nu\rho}$, and $E_{\mu\nu}$ *depend only on the undifferentiated fields*, with $D^{\mu\nu}$, $E^{\mu\nu\rho}$, and $E_{\mu\nu}$ completely antisymmetric. In order to analyze the structure of these functions, it is convenient to introduce an operator N that counts all the fields excepting φ

$$\bar{\Phi}^{\alpha_0} = \left(H^\mu, A_\mu, B^{\mu\nu}, \phi_{\mu\nu}, K^{\mu\nu\rho} \right) , \tag{D.19}$$

and their derivatives, defined through

$$N = \sum_{k \geq 0} \left(\partial_{\mu_1 \cdots \mu_k} \bar{\Phi}^{\alpha_0} \right) \frac{\partial}{\partial \left(\partial_{\mu_1 \cdots \mu_k} \bar{\Phi}^{\alpha_0} \right)} \,. \tag{D.20}$$

Then, it is simple to see that, for every non-integrated density χ depending on $\bar{\Phi}^{\alpha_0}$, their derivatives, and *the undifferentiated scalar field* φ, we have that

$$N\chi = \bar{\Phi}^{\alpha_0} \frac{\delta \chi}{\delta \bar{\Phi}^{\alpha_0}} + \partial^\mu j_\mu \,, \tag{D.21}$$

where j_μ are some local currents. Let $\chi^{(k)}$ be a homogeneous polynomial of order $k > 0$ in the fields $\bar{\Phi}^{\alpha_0}$ and their derivatives. Then, it follows that

$$N\chi^{(k)} = k\chi^{(k)} \,. \tag{D.22}$$

Using the solutions (D.17) and (D.18) in (D.21), we get that

$$N\omega_1 = -D\partial_\mu H^\mu + \frac{1}{2} D^{\mu\nu} \partial_{[\mu} A_{\nu]} - 2D_\mu \partial_\nu B^{\nu\mu} - \frac{1}{3} E^{\mu\nu\rho} \partial_{[\mu} \phi_{\nu\rho]} - 3E_{\mu\nu} \partial_\rho K^{\mu\nu\rho} + \partial^\mu v_\mu \,. \tag{D.23}$$

At this stage, we decompose ω_1 under the form

$$\omega_1 = \sum_{k>0} \omega_1^{(k)} \,, \tag{D.24}$$

[the value $k = 0$ is excluded due to the fact that we work with ω_1 like in (D.13)], where N acts on the component $\omega_1^{(k)}$ via

$$N\omega_1^{(k)} = k\omega_1^{(k)} \,. \tag{D.25}$$

Substituting (D.25) in the expansion (D.24), we infer that

$$N\omega_1 = \sum_{k>0} k\omega_1^{(k)} \,. \tag{D.26}$$

Comparing (D.23) with (D.26), we conclude that the decomposition (D.24) induces a similar decomposition with respect to the functions D, D_μ, $D^{\mu\nu}$, $E_{\mu\nu}$, and $E^{\mu\nu\rho}$, i.e.

$$D = \sum_{k>0} D^{(k-1)}\,, \qquad D_\mu = \sum_{k>0} D_\mu^{(k-1)}\,, \qquad D^{\mu\nu} = \sum_{k>0} D^{(k-1)\mu\nu}\,, \tag{D.27}$$

$$E_{\mu\nu} = \sum_{k>0} E_{\mu\nu}^{(k-1)}\,, \qquad E^{\mu\nu\rho} = \sum_{k>0} E^{(k-1)\mu\nu\rho}\,. \tag{D.28}$$

Inserting now the outcomes (D.27) and (D.28) in (D.23) and then comparing the corresponding result with (D.26), we deduce that

$$\omega_1^{(k)} = -\frac{1}{k}\Big(D^{(k-1)}\partial_\mu H^\mu - \frac{1}{2}D^{(k-1)\mu\nu}\partial_{[\mu}A_{\nu]} + 2D_\mu^{(k-1)}\partial_\nu B^{\nu\mu} + + \frac{1}{3}E^{(k-1)\mu\nu\rho}\partial_{[\mu}\phi_{\nu\rho]} + 3E_{\mu\nu}^{(k-1)}\partial_\rho K^{\mu\nu\rho}\Big) + \partial^\mu v_\mu^{(k)}. \tag{D.29}$$

Putting together the relations (D.29) for the various values of k like in (D.24), we are able to reconstruct ω_1 like

$$\omega_1 = -\bar{D}\partial_\mu H^\mu + \frac{1}{2}\bar{D}^{\mu\nu}\partial_{[\mu}A_{\nu]} - 2\bar{D}_\mu\partial_\nu B^{\nu\mu} - \frac{1}{3}\bar{E}^{\mu\nu\rho}\partial_{[\mu}\phi_{\nu\rho]} - 3\bar{E}_{\mu\nu}\partial_\rho K^{\mu\nu\rho} + \partial^\mu \bar{v}_\mu\,, \tag{D.30}$$

where

$$\bar{D} = \sum_{k>0}\frac{1}{k}D^{(k-1)}\,, \qquad \bar{D}_\mu = \sum_{k>0}\frac{1}{k}D_\mu^{(k-1)}\,, \qquad \bar{D}^{\mu\nu} = \sum_{k>0}\frac{1}{k}D^{(k-1)\mu\nu}\,, \tag{D.31}$$

$$\bar{E}_{\mu\nu} = \sum_{k>0}\frac{1}{k}E_{\mu\nu}^{(k-1)}\,, \qquad \bar{E}^{\mu\nu\rho} = \sum_{k>0}\frac{1}{k}E^{(k-1)\mu\nu\rho}\,. \tag{D.32}$$

It is clear from the definitions of δ that (D.30) is in fact δ-exact modulo d

$$\omega_1 = \delta\left[-\left(\bar{D}\varphi^* + \bar{D}^{\mu\nu}B_{\mu\nu}^* + 2\bar{D}_\mu A^{*\mu} - \bar{E}^{\mu\nu\rho}K_{\mu\nu\rho}^* + 3\bar{E}_{\mu\nu}\phi^{*\mu\nu}\right)\right] + \partial^\mu \bar{v}_\mu\,. \tag{D.33}$$

By virtue of the above discussion on trivial interactions, we can state that ω_1 is trivial, and therefore we can take $\omega_1 = 0$.

In conclusion, the general, nontrivial, consistent solution to the equation (D.1) takes the simple form

$$\bar{a}_0 = \bar{M}(\varphi)\,. \tag{D.34}$$

E. Some notations made in the body of the paper

The various notations used in (4.34) are listed below. Thus, the objects denoted by $X^{(0)}$ and $\left(X_a^{(1)}\right)_{a=\overline{0,5}}$ are expressed by

$$X^{(0)} = -2\eta\,, \tag{E.1}$$

$$X_0^{(1)} = -4\left[3\left(\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}C^{\mu\nu\rho\lambda\sigma} + \mathcal{G}^*_{\mu\nu\rho\lambda}C^{\mu\nu\rho\lambda} + K^*_{\mu\nu\rho}C^{\mu\nu\rho}\right) + \phi_{\mu\nu}C^{\mu\nu}\right], \tag{E.2}$$

$$X_1^{(1)} = 4\Big[\left(C^*_{\mu\nu\rho\lambda\sigma}C - C^*_{[\mu\nu\rho\lambda}C_{\sigma]} - C^*_{[\mu\nu\rho}\phi_{\lambda\sigma]} - 3C^*_{[\mu\nu}K^*_{\rho\lambda\sigma]} - 3H^*_{[\mu}\mathcal{G}^*_{\nu\rho\lambda\sigma]}\right)C^{\mu\nu\rho\lambda\sigma} + \\ + \left(C^*_{\mu\nu\rho\lambda}C - C^*_{[\mu\nu\rho}C_{\lambda]} - C^*_{[\mu\nu}\phi_{\rho\lambda]} - 3H^*_{[\mu}K^*_{\nu\rho\lambda]}\right)C^{\mu\nu\rho\lambda} + \\ + \left(C^*_{\mu\nu\rho}C - C^*_{[\mu\nu}C_{\rho]} - H^*_{[\mu}\phi_{\nu\rho]}\right)C^{\mu\nu\rho} + \left(C^*_{\mu\nu}C - H^*_{[\mu}C_{\nu]}\right)C^{\mu\nu}\Big], \tag{E.3}$$

$$X_2^{(1)} = 4\Big\{\Big[\left(H^*_{[\mu}C^*_{\nu\rho\lambda\sigma]} + C^*_{[\mu\nu}C^*_{\rho\lambda\sigma]}\right)C - 5\left(H^*_{[\mu}C^*_{\nu\rho\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]}\right)C_\sigma - \\ - H^*_{[\mu}C^*_{\nu\rho}\phi_{\lambda\sigma]} - 3H^*_{[\mu}H^*_{\nu}K^*_{\rho\lambda\sigma]}\Big]C^{\mu\nu\rho\lambda\sigma} + \\ + \left[\left(H^*_{[\mu}C^*_{\nu\rho\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]}\right)C - H^*_{[\mu}C^*_{\nu\rho}C_{\lambda]} - H^*_{[\mu}H^*_{\nu}\phi_{\rho\lambda]}\right]C^{\mu\nu\rho\lambda} + \\ + \left(H^*_{[\mu}C^*_{\nu\rho]}C - H^*_{[\mu}H^*_{\nu}C_{\rho]}\right)C^{\mu\nu\rho} + H^*_{\mu}H^*_{\nu}CC^{\mu\nu}\Big\}, \tag{E.4}$$

$$X_3^{(1)} = 4\Big\{\Big[\left(H^*_{[\mu}H^*_{\nu}C^*_{\rho\lambda\sigma]} + H^*_{[\mu}C^*_{\nu\rho}C^*_{\lambda\sigma]}\right)C - H^*_{[\mu}H^*_{\nu}C^*_{\rho\lambda}C_{\sigma]} - \\ - H^*_{[\mu}H^*_{\nu}H^*_{\rho}\phi_{\lambda\sigma]}\Big]C^{\mu\nu\rho\lambda\sigma} + \left(H^*_{[\mu}H^*_{\nu}C^*_{\rho\lambda]}C - H^*_{[\mu}H^*_{\nu}H^*_{\rho}C_{\lambda]}\right)C^{\mu\nu\rho\lambda} + \\ + H^*_{[\mu}H^*_{\nu}H^*_{\rho]}CC^{\mu\nu\rho}\Big\}, \tag{E.5}$$

$$X_4^{(1)} = 4H^*_{\mu}H^*_{\nu}H^*_{\rho}\left[5\left(2C^*_{\lambda\sigma}C - H^*_{\lambda}C_{\sigma}\right)C^{\mu\nu\rho\lambda\sigma} + H^*_{\lambda}CC^{\mu\nu\rho\lambda}\right], \tag{E.6}$$

$$X_5^{(1)} = 4H^*_{\mu}H^*_{\nu}H^*_{\rho}H^*_{\lambda}H^*_{\sigma}CC^{\mu\nu\rho\lambda\sigma}. \tag{E.7}$$

The notations $\left(X_a^{(2)}\right)_{a=\overline{0,5}}$ signify the functions

$$X_0^{(2)} = 4\Big\{\Big(\eta^*_{\mu\nu\rho\lambda\sigma}C + \eta^*_{[\mu\nu\rho\lambda}C_{\sigma]} - \eta^*_{[\mu\nu\rho}\phi_{\lambda\sigma]} + 3B^*_{[\mu\nu}K^*_{\rho\lambda\sigma]} - \\ - \frac{3}{2}A_{[\mu}\mathcal{G}^*_{\nu\rho\lambda\sigma]} + \frac{1}{2}\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}\eta\Big)\eta^{\mu\nu\rho\lambda\sigma} + \\ + \left[\eta^*_{\mu\nu\rho\lambda}C + \eta^*_{[\mu\nu\rho}C_{\lambda]} - B^*_{[\mu\nu}\phi_{\rho\lambda]} + \frac{3}{2}\left(A_{[\mu}K^*_{\nu\rho\lambda]} - \mathcal{G}^*_{\mu\nu\rho\lambda}\eta\right)\right]\eta^{\mu\nu\rho\lambda} + \\ + \left(\eta^*_{\mu\nu\rho}C + B^*_{[\mu\nu}C_{\rho]} - \frac{1}{2}A_{[\mu}\phi_{\nu\rho]} - \frac{3}{2}K^*_{\mu\nu\rho}\eta\right)\eta^{\mu\nu\rho} - \eta^*\eta C - \\ - A^*_{\mu}\left(A^{\mu}C + \eta C^{\mu}\right) + \left(B^*_{\mu\nu}C + \frac{1}{2}A_{[\mu}C_{\nu]} - \frac{1}{2}\phi_{\mu\nu}\eta\right)B^{\mu\nu}\Big\}, \tag{E.8}$$

$$X_1^{(2)} = 2\Big\{\Big[C^*_{\mu\nu\rho\lambda\sigma}\eta C + 5C^*_{\mu\nu\rho\lambda}\left(A_\sigma C + \eta C_\sigma\right) + 20C^*_{\mu\nu\rho}\left(B^*_{\lambda\sigma}C + A_\lambda C_\sigma - \frac{1}{2}\phi_{\lambda\sigma}\eta\right) + \\ + 20C^*_{\mu\nu}\left(\eta^*_{\rho\lambda\sigma}C + B^*_{[\rho\lambda}C_{\sigma]} - \frac{1}{2}A_{[\rho}\phi_{\lambda\sigma]} - \frac{3}{2}K^*_{\rho\lambda\sigma}\eta\right) + \\ + 10H^*_{\mu}\left(\eta^*_{\nu\rho\lambda\sigma}C + \eta^*_{[\nu\rho\lambda}C_{\sigma]} - B^*_{[\nu\rho}\phi_{\lambda\sigma]} + 3A_{[\nu}K^*_{\rho\lambda\sigma]} - \frac{3}{2}\mathcal{G}^*_{\nu\rho\lambda\sigma}\eta\right)\Big]\eta^{\mu\nu\rho\lambda\sigma} + \\ + \Big[C^*_{\mu\nu\rho\lambda}\eta C + 4C^*_{\mu\nu\rho}\left(A_\lambda C + \eta C_\lambda\right) + 12C^*_{\mu\nu}\left(B^*_{\rho\lambda}C + A_{[\rho}C_{\lambda]} - \frac{1}{2}\eta\phi_{\rho\lambda}\right) + \\ + 8H^*_{\mu}\left(\eta^*_{\nu\rho\lambda}C + B^*_{[\nu\rho}C_{\lambda]} - \frac{1}{2}A_{[\nu}\phi_{\rho\lambda]} - \frac{3}{2}K^*_{\nu\rho\lambda}\eta\right)\Big]\eta^{\mu\nu\rho\lambda} +$$

$$+\left[C^*_{\mu\nu\rho}\eta C+3C^*_{\mu\nu}\left(A_\rho C+\eta C_\rho\right)+6H^*_\mu\left(B^*_{\nu\rho}C+\frac{1}{2}\left(A_{[\nu}C_{\rho]}-\phi_{\nu\rho}\eta\right)\right)\right]\eta^{\mu\nu\rho}+$$
$$+\left[C^*_{\mu\nu}\eta C+2H^*_\mu\left(A_\nu C+\eta C_\nu\right)\right]B^{\mu\nu}+2H^*_\mu A^{*\mu}\eta C\bigg\}, \tag{E.9}$$

$$X^{(2)}_2=2\bigg\{\bigg[\left(H^*_{[\mu}C^*_{\nu\rho\lambda\sigma]}+C^*_{[\mu\nu}C^*_{\rho\lambda\sigma]}\right)\eta C+5\left(H^*_{[\mu}C^*_{\nu\rho\lambda]}+C^*_{[\mu\nu}C^*_{\rho\lambda]}\right)\left(A_\sigma C+\eta C_\sigma\right)+$$
$$+20H^*_{[\mu}C^*_{\nu\rho]}\left(B^*_{\lambda\sigma}C+\frac{1}{2}\left(A_{[\lambda}C_{\sigma]}-\phi_{\lambda\sigma}\eta\right)\right)+$$
$$+20H^*_\mu H^*_\nu\left(\eta^*_{\rho\lambda\sigma}C+B^*_{[\rho\lambda}C_{\sigma]}-\frac{1}{2}\left(A_{[\rho}\phi_{\lambda\sigma]}+3K^*_{\rho\lambda\sigma}\eta\right)\right)\bigg]\eta^{\mu\nu\rho\lambda\sigma}+$$
$$+\bigg[\left(H^*_{[\mu}C^*_{\nu\rho\lambda]}+C^*_{[\mu\nu}C^*_{\rho\lambda]}\right)\eta C+4H^*_{[\mu}C^*_{\nu\rho]}\left(A_\lambda C+\eta C_\lambda\right)+$$
$$+12H^*_\mu H^*_\nu\left(\frac{1}{2}\left(A_{[\rho}C_{\lambda]}-\phi_{\rho\lambda}\eta\right)+B^*_{\rho\lambda}C\right)\bigg]\eta^{\mu\nu\rho\lambda}+$$
$$+\left[H^*_{[\mu}C^*_{\nu\rho]}\eta C+3H^*_\mu H^*_\nu\left(A_\rho C+\eta C_\rho\right)\right]\eta^{\mu\nu\rho}+H^*_\mu H^*_\nu B^{\mu\nu}\eta C\bigg\}, \tag{E.10}$$

$$X^{(2)}_3=2\bigg\{\bigg[\left(H^*_{[\mu}H^*_\nu C^*_{\rho\lambda\sigma]}+H^*_{[\mu}C^*_{\nu\rho}C^*_{\lambda\sigma]}\right)\eta C+5H^*_{[\mu}H^*_\nu C^*_{\rho\lambda]}\left(A_\sigma C+\eta C_\sigma\right)+$$
$$+20H^*_\mu H^*_\nu H^*_\rho\left(\frac{1}{2}\left(A_{[\lambda}C_{\sigma]}-\phi_{\lambda\sigma}\eta\right)+B^*_{\lambda\sigma}C\right)\bigg]\eta^{\mu\nu\rho\lambda\sigma}+$$
$$+\left[H^*_{[\mu}H^*_\nu C^*_{\rho\lambda]}\eta C+4H^*_\mu H^*_\nu H^*_\rho\left(A_\lambda C+\eta C_\lambda\right)\right]\eta^{\mu\nu\rho\lambda}+$$
$$+H^*_\mu H^*_\nu H^*_\rho\eta C\eta^{\mu\nu\rho}\bigg\}, \tag{E.11}$$

$$X^{(2)}_4=20H^*_\mu H^*_\nu H^*_\rho\left[\left(C^*_{\lambda\sigma}\eta C+\frac{1}{2}H^*_\lambda\left(A_\sigma C+\eta C_\sigma\right)\right)\eta^{\mu\nu\rho\lambda\sigma}+\frac{1}{10}H^*_\lambda\eta C\eta^{\mu\nu\rho\lambda}\right], \tag{E.12}$$

$$X^{(2)}_5=2H^*_\mu H^*_\nu H^*_\rho H^*_\lambda H^*_\sigma\eta C\eta^{\mu\nu\rho\lambda\sigma}. \tag{E.13}$$

The elements $\left(X^{(3)}_a\right)_{a=\overline{0,5}}$ read as

$$X^{(3)}_0=12\bigg[\left(C^*C+2C^*_\mu C^\mu+2\phi^*_{\mu\nu}\phi^{\mu\nu}-2K^*_{\mu\nu\rho}K^{\mu\nu\rho}-2\mathcal{G}^*_{\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\right)C+$$
$$+2\left(\mathcal{G}^*_{[\mu\nu\rho\lambda}C_{\sigma]}+K^*_{[\mu\nu\rho}\phi_{\lambda\sigma]}-\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}C\right)\mathcal{G}^{\mu\nu\rho\lambda\sigma}+$$
$$+2\left(K^*_{[\mu\nu\rho}C_{\lambda]}+\frac{1}{3}\phi_{[\mu\nu}\phi_{\lambda\rho]}\right)\mathcal{G}^{\mu\nu\rho\lambda}+2\left(-\phi^{*\mu\nu}C_\mu C_\nu+K^{\mu\nu\rho}\phi_{\mu\nu}C_\rho\right)\bigg], \tag{E.14}$$

$$X^{(3)}_1=4\bigg\{\bigg[\left(C^*_{\mu\nu\rho\lambda\sigma}C-2C^*_{[\mu\nu\rho\lambda}C_{\sigma]}\right)C-20C^*_{\mu\nu\rho}\left(\phi_{\lambda\sigma}C-C_\lambda C_\sigma\right)-$$
$$-60C^*_{\mu\nu}\left(K^*_{\rho\lambda\sigma}C-\frac{1}{3}\phi_{[\rho\lambda}C_{\sigma]}\right)+$$
$$+30H^*_\mu\left(K^*_{[\nu\rho\lambda}C_{\sigma]}-\mathcal{G}^*_{\nu\rho\lambda\sigma}C+\frac{1}{3}\phi_{[\nu\rho}\phi_{\lambda\sigma]}\right)\bigg]\mathcal{G}^{\mu\nu\rho\lambda\sigma}+$$

$$+\left[\left(C^*_{\mu\nu\rho\lambda}C-2C^*_{[\mu\nu\rho}C_{\lambda]}\right)C+12C^*_{\mu\nu}\left(C_\rho C_\lambda-\phi_{\rho\lambda}C\right)-\right.$$
$$\left.-12H^*_\mu\left(K^*_{\nu\rho\lambda}C-\frac{2}{3}\phi_{[\nu\rho}C_{\lambda]}\right)\right]\mathcal{G}^{\mu\nu\rho\lambda}+$$
$$+\left[\left(C^*_{\mu\nu\rho}C-2C^*_{[\mu\nu}C_{\rho]}\right)C-3H^*_\mu\left(\phi_{\nu\rho}C-4C_\nu C_\rho\right)\right]K^{\mu\nu\rho}-$$
$$\left.-3\left[C^*_{\mu\nu}\phi^{*\mu\nu}C+H^*_\mu\left(C^{*\mu}C+2\phi^{*\mu\nu}C_\nu\right)\right]C\right\}, \tag{E.15}$$

$$X^{(3)}_2=4\left\{\left[\left(H^*_{[\mu}C^*_{\nu\rho\lambda\sigma]}+C^*_{[\mu\nu}C^*_{\rho\lambda\sigma]}\right)CC-10\left(H^*_{[\mu}C^*_{\nu\rho\lambda]}+C^*_{[\mu\nu}C^*_{\rho\lambda]}\right)C_\sigma C-\right.\right.$$
$$\left.-20H^*_{[\mu}C^*_{\nu\rho]}\left(\phi_{\lambda\sigma}C-C_\lambda C_\sigma\right)-60H^*_\mu H^*_\nu\left(K^*_{\rho\lambda\sigma}C-\frac{1}{3}C_{[\rho}\phi_{\lambda\sigma]}\right)\right]\mathcal{G}^{\mu\nu\rho\lambda\sigma}+$$
$$+\left[\left(H^*_{[\mu}C^*_{\nu\rho\lambda]}+C^*_{[\mu\nu}C^*_{\rho\lambda]}\right)CC-12H^*_{[\mu}C^*_{\nu\rho]}C_\lambda C+\right.$$
$$\left.+12H^*_\mu H^*_\nu\left(C_\rho C_\lambda-\phi_{\rho\lambda}C\right)\right]\mathcal{G}^{\mu\nu\rho\lambda}+$$
$$\left.+\left(H^*_{[\mu}C^*_{\nu\rho]}C-6H^*_\mu H^*_\nu C_\rho\right)K^{\mu\nu\rho}C-3H^*_\mu H^*_\nu\phi^{*\mu\nu}CC\right\}, \tag{E.16}$$

$$X^{(3)}_3=4\left\{\left[\left(H^*_{[\mu}H^*_\nu C^*_{\rho\lambda\sigma]}+H^*_{[\mu}C^*_{\nu\rho}C^*_{\lambda\sigma]}\right)C-2H^*_{[\mu}H^*_\nu C^*_{\rho\lambda}C_{\sigma]}-\right.\right.$$
$$\left.-20H^*_\mu H^*_\nu H^*_\rho\left(\phi_{\lambda\sigma}C-C_\lambda C_\sigma\right)\right]\mathcal{G}^{\mu\nu\rho\lambda\sigma}+$$
$$\left.+H^*_{[\mu}H^*_\nu C^*_{\rho\lambda]}\mathcal{G}^{\mu\nu\rho\lambda}CC+H^*_\mu H^*_\nu H^*_\rho\left(K^{\mu\nu\rho}C-8C_\lambda\mathcal{G}^{\mu\nu\rho\lambda}\right)C\right\}, \tag{E.17}$$

$$X^{(3)}_4=4\left[H^*_{[\mu}H^*_\nu H^*_\rho C^*_{\lambda\sigma]}CC\mathcal{G}^{\mu\nu\rho\lambda\sigma}+H^*_\mu H^*_\nu H^*_\rho H^*_\lambda C\left(C\mathcal{G}^{\mu\nu\rho\lambda}-10C_\sigma\mathcal{G}^{\mu\nu\rho\lambda\sigma}\right)\right], \tag{E.18}$$

$$X^{(3)}_5=4H^*_\mu H^*_\nu H^*_\rho H^*_\lambda H^*_\sigma CC\mathcal{G}^{\mu\nu\rho\lambda\sigma}\,. \tag{E.19}$$

The quantities $\left(X^{(4)}_a\right)_{a=\overline{0,5}}$ mean

$$X^{(4)}_0=12\epsilon^{\alpha\beta\gamma\delta\varepsilon}\left[-\frac{1}{3}\left(\eta^*_{\mu\nu\rho\lambda\sigma}\eta^{\mu\nu\rho\lambda\sigma}+\eta^*_{\mu\nu\rho\lambda}\eta^{\mu\nu\rho\lambda}+\eta^*_{\mu\nu\rho}\eta^{\mu\nu\rho}-\right.\right.$$
$$\left.-\eta^*\eta+B^*_{\mu\nu}B^{\mu\nu}-A^*_\mu A^\mu\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}+$$
$$+\left(\frac{1}{3}\eta^*_{\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}-\frac{1}{3}\eta^*_{\mu\nu\rho}K^{\mu\nu\rho}-\right.$$
$$\left.-\frac{1}{2}C^*\eta+\frac{1}{2}C^{*\mu}A_\mu-B^*_{\mu\nu}\phi^{*\mu\nu}\right)\eta_{\alpha\beta\gamma\delta\varepsilon}+$$
$$+\left(\frac{4}{3}\eta^*_{\mu\nu\rho}\eta^{\mu\nu\rho\lambda}-B^*_{\mu\nu}\eta^{\mu\nu\lambda}+\frac{1}{3}A_\mu B^{\mu\lambda}-\frac{1}{3}A^{*\lambda}\eta\right)\sigma_{\lambda\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon}+$$
$$+\left(\frac{1}{2}C^{*\rho}\eta-B^*_{\mu\nu}K^{\mu\nu\rho}+A_\mu\phi^{*\mu\rho}\right)\sigma_{\rho\alpha}\eta_{\beta\gamma\delta\varepsilon}+$$
$$\left.+\frac{1}{2}\left(\phi^{*\nu\rho}\eta-A_\mu K^{\mu\nu\rho}\right)\sigma_{\nu\alpha}\sigma_{\rho\beta}\eta_{\gamma\delta\varepsilon}-\frac{5}{3}B_{[\alpha\beta}K_{\gamma\delta\varepsilon]}\eta\right], \tag{E.20}$$

$$
\begin{aligned}
X_1^{(4)} = 12\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Bigg[\frac{1}{3}\Bigg(& - H^*_{[\mu}\eta^*_{\nu\rho\lambda\sigma]}\eta^{\mu\nu\rho\lambda\sigma} - H^*_{[\mu}\eta^*_{\nu\rho\lambda]}\eta^{\mu\nu\rho\lambda} - \\
& - H^*_{[\mu}B^*_{\nu\rho]}\eta^{\mu\nu\rho} - H^*_\mu A^{*\mu}\eta - \frac{1}{2}H^*_{[\mu}A_{\nu]}B^{\mu\nu}\Bigg)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
& + \frac{1}{3}\left(-\frac{3}{2}H^*_{[\mu}A_{\nu]}\eta^{\mu\nu\rho} + H^*_\mu B^{\mu\rho}\eta\right)\sigma_{\rho\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon} + \\
& + \frac{1}{3}\Bigg(H^*_{[\mu}\eta^*_{\nu\rho\lambda]}\eta^{\mu\nu\rho\lambda} - H^*_{[\mu}B^*_{\nu\rho]}K^{\mu\nu\rho} - \\
& \qquad - \frac{3}{2}H^*_{[\mu}A_{\nu]}\phi^{*\mu\nu} + \frac{3}{2}H^*_\mu C^{*\mu}\eta\Bigg)\eta_{\alpha\beta\gamma\delta\varepsilon} + \\
& + 2\left(-\frac{2}{3}H^*_{[\mu}B^*_{\nu\rho]}\mathcal{G}^{\mu\nu\rho\lambda} - \frac{1}{4}H^*_{[\mu}A_{\nu]}K^{\mu\nu\lambda} - H^*_\mu\phi^{*\mu\lambda}\eta\right)\sigma_{\lambda\alpha}\eta_{\beta\gamma\delta\varepsilon} + \\
& + \frac{1}{2}H^*_\mu K^{\mu\nu\rho}\sigma_{\nu\alpha}\sigma_{\rho\beta}\eta\eta_{\gamma\delta\varepsilon}\Bigg],
\end{aligned}
\tag{E.21}
$$

$$
\begin{aligned}
X_2^{(4)} = 12\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Bigg\{ & -\frac{1}{3}\Bigg[\left(C^*_{[\mu\nu}\eta^*_{\rho\lambda\sigma]} + H^*_{[\mu}H^*_\nu\eta^*_{\rho\lambda\sigma]}\right)\eta^{\mu\nu\rho\lambda\sigma} + \\
& \qquad + \left(C^*_{[\mu\nu}B^*_{\rho\lambda]} + H^*_{[\mu}H^*_\nu B^*_{\rho\lambda]}\right)\eta^{\mu\nu\rho\lambda} + \\
& \qquad + \frac{1}{2}\left(C^*_{[\mu\nu}A_{\rho]} + H^*_{[\mu}H^*_\nu A_{\rho]}\right)\eta^{\mu\nu\rho} + \\
& \qquad + \frac{1}{2}\left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)B^{\mu\nu}\eta\Bigg]\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
& + \frac{1}{2}\Bigg[\frac{4}{3}\left(C^*_{[\mu\nu}A_{\rho]} + H^*_{[\mu}H^*_\nu A_{\rho]}\right)\eta^{\mu\nu\rho\lambda} + \\
& \qquad + \left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)\eta^{\mu\nu\lambda}\eta\Bigg]\sigma_{\lambda\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon} + \\
& + \frac{1}{6}\Bigg[\frac{1}{2}\left(C^*_{[\mu\nu}B^*_{\rho\lambda]} + H^*_{[\mu}H^*_\nu B^*_{\rho\lambda]}\right)\mathcal{G}^{\mu\nu\rho\lambda} - \\
& \qquad - \left(C^*_{[\mu\nu}A_{\rho]} + H^*_{[\mu}H^*_\nu A_{\rho]}\right)K^{\mu\nu\rho} + \\
& \qquad + \frac{1}{3}\left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)\phi^{*\mu\nu}\eta\Bigg]\eta_{\alpha\beta\gamma\delta\varepsilon} - \\
& - \frac{1}{2}\left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)K^{\mu\nu\rho}\sigma_{\rho\alpha}\eta\eta_{\beta\gamma\delta\varepsilon}\Bigg\},
\end{aligned}
\tag{E.22}
$$

$$
\begin{aligned}
X_3^{(4)} = 4\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Bigg\{ & -\Bigg[\left(C^*_{[\mu\nu\rho}B^*_{\lambda\sigma]} + C^*_{[\mu\nu}H^*_\rho B^*_{\lambda\sigma]} + H^*_{[\mu}H^*_\nu H^*_\rho B^*_{\lambda\sigma]}\right)\eta^{\mu\nu\rho\lambda\sigma} + \\
& \quad + \frac{1}{2}\left(C^*_{[\mu\nu}H^*_\rho A_{\lambda]} + H^*_{[\mu}H^*_\nu H^*_\rho A_{\lambda]} + C^*_{[\mu\nu\rho}A_{\lambda]}\right)\eta^{\mu\nu\rho\lambda} - \\
& \quad - \frac{1}{2}\left(C^*_{\mu\nu\rho} + C^*_{[\mu\nu}H^*_{\rho]} + H^*_\mu H^*_\nu H^*_\rho\right)\eta^{\mu\nu\rho}\eta\Bigg]\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
& + \frac{1}{2}\Bigg[\left(C^*_{[\mu\nu\rho}A_{\lambda]} + C^*_{[\mu\nu}H^*_\rho A_{\lambda]} + H^*_{[\mu}H^*_\nu H^*_\rho A_{\lambda]}\right)\mathcal{G}^{\mu\nu\rho\lambda} - \\
& \quad - \left(C^*_{\mu\nu\rho} + C^*_{[\mu\nu}H^*_{\rho]} + H^*_\mu H^*_\nu H^*_\rho\right)K^{\mu\nu\rho}\eta\Bigg]\eta_{\alpha\beta\gamma\delta\varepsilon} +
\end{aligned}
$$

$$+ 2\left(C^*_{\mu\nu\rho} + C^*_{[\mu\nu}H^*_{\rho]} + H^*_\mu H^*_\nu H^*_\rho\right)\eta^{\mu\nu\rho\lambda}\sigma_{\lambda\alpha}\eta\mathcal{G}_{\beta\gamma\delta\varepsilon}\Big\}, \tag{E.23}$$

$$\begin{aligned} X_4^{(4)} = & -2\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big\{\Big[\Big(C^*_{[\mu\nu\rho\lambda}A_{\sigma]} + C^*_{[\mu\nu\rho}H^*_\lambda A_{\sigma]} + C^*_{[\mu\nu}C^*_{\rho\lambda}A_{\sigma]} + \\ & + C^*_{[\mu\nu}H^*_\rho H^*_\lambda A_{\sigma]} + H^*_{[\mu}H^*_\nu H^*_\rho H^*_\lambda A_{\sigma]}\Big)\eta^{\mu\nu\rho\lambda\sigma} + \\ & + \Big(C^*_{\mu\nu\rho\lambda} + C^*_{[\mu\nu\rho}H^*_{\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]} + \\ & + C^*_{[\mu\nu}H^*_\rho H^*_{\lambda]} + H^*_\mu H^*_\nu H^*_\rho H^*_\lambda\Big)\eta^{\mu\nu\rho\lambda}\Big]\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\ & + \Big(C^*_{\mu\nu\rho\lambda} + C^*_{[\mu\nu\rho}H^*_{\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]} + \\ & + C^*_{[\mu\nu}H^*_\rho H^*_{\lambda]} + H^*_\mu H^*_\nu H^*_\rho H^*_\lambda\Big)\mathcal{G}^{\mu\nu\rho\lambda}\eta\eta_{\alpha\beta\gamma\delta\varepsilon}\Big\}, \end{aligned} \tag{E.24}$$

$$\begin{aligned} X_5^{(4)} = & \; 2\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(C^*_{\mu\nu\rho\lambda\sigma} + C^*_{[\mu\nu\rho\lambda}H^*_{\sigma]} + C^*_{[\mu\nu\rho}C^*_{\lambda\sigma]} + C^*_{[\mu\nu\rho}H^*_\lambda H^*_{\sigma]} + \\ & + C^*_{[\mu\nu}C^*_{\rho\lambda}H^*_{\sigma]} + C^*_{[\mu\nu}H^*_\rho H^*_\lambda H^*_{\sigma]} + \\ & + H^*_\mu H^*_\nu H^*_\rho H^*_\lambda H^*_\sigma\Big)\eta^{\mu\nu\rho\lambda\sigma}\eta\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}. \end{aligned} \tag{E.25}$$

The components $\left(X_a^{(5)}\right)_{a=\overline{0,5}}$ are given by

$$\begin{aligned} X_0^{(5)} = & \; 24\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(-\frac{1}{2}C^*C_{\alpha\beta\gamma\delta\varepsilon} + \frac{1}{10}C^*_{[\alpha}C_{\beta\gamma\delta\varepsilon]} + \frac{1}{20}\phi^*_{[\alpha\beta}C_{\gamma\delta\varepsilon]} - \\ & - \frac{1}{60}K_{[\alpha\beta\gamma}C_{\delta\varepsilon]} - \frac{1}{30}H_{[\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon]} + \frac{1}{6}\varphi^*\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\Big), \end{aligned} \tag{E.26}$$

$$\begin{aligned} X_1^{(5)} = & \; 12\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[\Big(H^*_\mu C^{*\mu}C_{\alpha\beta\gamma\delta\varepsilon} - \frac{1}{3}H^*_\mu H^\mu\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\Big) + \\ & + 2\sigma_{\rho\alpha}\Big(-H^*_\mu\phi^{*\mu\rho}C_{\beta\gamma\delta\varepsilon} + \frac{1}{3}H^*_\mu C^{\mu\rho}\mathcal{G}_{\beta\gamma\delta\varepsilon}\Big) - \\ & - H^{*\mu}C_{\mu\alpha\beta}K_{\gamma\delta\varepsilon}\Big], \end{aligned} \tag{E.27}$$

$$\begin{aligned} X_2^{(5)} = & \; 12\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[\left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)\Big(\phi^{*\mu\nu}C_{\alpha\beta\gamma\delta\varepsilon} - \frac{1}{3}C^{\mu\nu}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\Big) - \\ & - \left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)\sigma_{\rho\alpha}\left(K^{\mu\nu\rho}C_{\beta\gamma\delta\varepsilon} + C^{\mu\nu\rho}\mathcal{G}_{\beta\gamma\delta\varepsilon}\right)\Big], \end{aligned} \tag{E.28}$$

$$\begin{aligned} X_3^{(5)} = & -4\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[\Big(C^*_{\mu\nu\rho} + C^*_{[\mu\nu}H^*_{\rho]} + H^*_\mu H^*_\nu H^*_\rho\Big)\times \\ & \times\left(K^{\mu\nu\rho}C_{\alpha\beta\gamma\delta\varepsilon} + C^{\mu\nu\rho}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\right) - \\ & - 4\Big(C^*_{\mu\nu\rho} + C^*_{[\mu\nu}H^*_{\rho]} + H^*_\mu H^*_\nu H^*_\rho\Big)C^{\mu\nu\rho\lambda}\sigma_{\lambda\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon}\Big], \end{aligned} \tag{E.29}$$

$$\begin{aligned} X_4^{(5)} = & -4\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(C^*_{\mu\nu\rho\lambda} + C^*_{[\mu\nu\rho}H^*_{\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]} + C^*_{[\mu\nu}H^*_\rho H^*_{\lambda]} + H^*_\mu H^*_\nu H^*_\rho H^*_\lambda\Big)\times \\ & \times\Big(\mathcal{G}^{\mu\nu\rho\lambda}C_{\alpha\beta\gamma\delta\varepsilon} + C^{\mu\nu\rho\lambda}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\Big), \end{aligned} \tag{E.30}$$

$$X_5^{(5)} = -4\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(C^*_{\mu\nu\rho\lambda\sigma} + C^*_{[\mu\nu\rho\lambda}H^*_{\sigma]} + C^*_{[\mu\nu\rho}C^*_{\lambda\sigma]} + C^*_{[\mu\nu\rho}H^*_\lambda H^*_{\sigma]} +$$

$$
\begin{aligned}
&+ C^*_{[\mu\nu}C^*_{\rho\lambda}H^*_{\sigma]} + C^*_{[\mu\nu}H^*_\rho H^*_\lambda H^*_{\sigma]} + \\
&+ H^*_\mu H^*_\nu H^*_\rho H^*_\lambda H^*_\sigma\Big)C^{\mu\nu\rho\lambda\sigma}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\,. \qquad \text{(E.31)}
\end{aligned}
$$

The terms denoted by $\left(X_a^{(6)}\right)_{a=\overline{0,5}}$ are of the form

$$
\begin{aligned}
X_0^{(6)} = 24\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[&\Big(2\mathcal{G}^*_{\mu\nu\rho\lambda\sigma}\mathcal{G}^{\mu\nu\rho\lambda\sigma} + \mathcal{G}^*_{\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda} - C^*C - \\
&\quad - C^*_\mu C^\mu + K^*_{\mu\nu\rho}K^{\mu\nu\rho} - \phi^*_{\mu\nu}\phi^{\mu\nu}\Big)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
&+ \left(C^{*\lambda}C + \phi_{\mu\nu}K^{\mu\nu\lambda} - 2\phi^{*\mu\lambda}C_\mu\right)\sigma_{\lambda\alpha}\mathcal{G}_{\beta\gamma\delta\varepsilon} - \\
&- \frac{1}{12}K^*_{\alpha\beta\gamma}\epsilon_{\delta\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\varepsilon\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'} + \\
&+ \phi^*_{\alpha\beta}K_{\gamma\delta\varepsilon}C - \frac{1}{24}C_\alpha\epsilon_{\beta\gamma\mu\nu\rho}K^{\mu\nu\rho}\epsilon_{\delta\varepsilon\mu'\nu'\rho'}K^{\mu'\nu'\rho'}\Big], \qquad \text{(E.32)}
\end{aligned}
$$

$$
\begin{aligned}
X_1^{(6)} = 24\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[&\Big(H^*_\mu C^{*\mu}C + 2H^*_\mu\phi^{*\mu\nu}C_\nu + \frac{1}{2}H^*_{[\mu}\mathcal{G}^*_{\nu\rho\lambda\sigma]}\mathcal{G}^{\mu\nu\rho\lambda\sigma} + \\
&\quad + \frac{1}{3}H^*_{[\mu}\phi_{\nu\rho]}K^{\mu\nu\rho} + H^*_{[\mu}K^*_{\nu\rho\lambda]}\mathcal{G}^{\mu\nu\rho\lambda}\Big)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
&+ \sigma_{\rho\alpha}\left(2H^*_\mu\phi^{*\mu\rho}C - H^*_{[\mu}C_{\nu]}K^{\mu\nu\rho}\right)\mathcal{G}_{\beta\gamma\delta\varepsilon} + \\
&+ \frac{1}{24}H^*_\alpha C\epsilon_{\beta\gamma\mu\nu\rho}K^{\mu\nu\rho}\epsilon_{\delta\varepsilon\mu'\nu'\rho'}K^{\mu'\nu'\rho'} - \\
&- \frac{1}{36}H^*_{[\alpha}\phi_{\beta\gamma]}\epsilon_{\delta\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\varepsilon\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'}\Big], \qquad \text{(E.33)}
\end{aligned}
$$

$$
\begin{aligned}
X_2^{(6)} = 24\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big\{&\Big[\frac{2}{3}\left(C^*_{[\mu\nu}C_{\rho]} + H^*_{[\mu}H^*_\nu C_{\rho]}\right)K^{\mu\nu\rho} + \left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)\phi^{*\mu\nu}C + \\
&\quad + \frac{1}{2}\left(C^*_{[\mu\nu}K^*_{\rho\lambda\sigma]} + H^*_{[\mu}H^*_\nu K^*_{\rho\lambda\sigma]}\right)\mathcal{G}^{\mu\nu\rho\lambda\sigma} + \\
&\quad + \frac{1}{3}\left(C^*_{[\mu\nu}\phi_{\rho\lambda]} + H^*_{[\mu}H^*_\nu\phi_{\rho\lambda]}\right)\mathcal{G}^{\mu\nu\rho\lambda}\Big]\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} - \\
&- \sigma_{\rho\alpha}\left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)K^{\mu\nu\rho}\mathcal{G}_{\beta\gamma\delta\varepsilon} - \qquad \text{(E.34)} \\
&- \frac{1}{6\cdot 4!}\left(C^*_{[\alpha\beta}C_{\gamma]} + H^*_{[\alpha}H^*_\beta C_{\gamma]}\right)\epsilon_{\delta\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\varepsilon\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
X_3^{(6)} = -8\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[&\left(C^*_{\mu\nu\rho} + C^*_{[\mu\nu}H^*_{\rho]} + H^*_\mu H^*_\nu H^*_\rho\right)K^{\mu\nu\rho}C\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
&+ \Big(C^*_{[\mu\nu\rho}A_{\lambda]} + C^*_{[\mu\nu}H^*_\rho A_{\lambda]} + \\
&\qquad + C^*_{[\mu\nu}H^*_\rho A_{\lambda]} + H^*_{[\mu}H^*_\nu H^*_\rho A_{\lambda]}\Big)\mathcal{G}^{\mu\nu\rho\lambda}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} - \\
&- \frac{1}{12}\left(C^*_{[\alpha\beta}C_{\gamma]} + H^*_{[\alpha}H^*_\beta C_{\gamma]}\right)\epsilon_{\delta\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\varepsilon\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'}\Big], \quad \text{(E.35)}
\end{aligned}
$$

$$
X_4^{(6)} = 4\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[\Big(C^*_{[\mu\nu\rho\lambda}C_{\sigma]} + C^*_{[\mu\nu\rho}H^*_\lambda C_{\sigma]} + C^*_{[\mu\nu}C^*_{\rho\lambda}C_{\sigma]} +
$$

$$
\begin{aligned}
&+ C^*_{[\mu\nu}H^*_\rho H^*_\lambda C_{\sigma]} + H^*_{[\mu}H^*_\nu H^*_\rho H^*_\lambda C_{\sigma]}\Big)\mathcal{G}^{\mu\nu\rho\lambda\sigma} - \\
&- \Big(C^*_{\mu\nu\rho\lambda} + C^*_{[\mu\nu\rho}H^*_{\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]} + \\
&\quad + C^*_{[\mu\nu}H^*_\rho H^*_{\lambda]} + H^*_\mu H^*_\nu H^*_\rho H^*_\lambda\Big)\mathcal{G}^{\mu\nu\rho\lambda}C\Big]\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\,,
\end{aligned} \tag{E.36}
$$

$$
\begin{aligned}
X^{(6)}_5 = -4\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(&C^*_{\mu\nu\rho\lambda\sigma} + C^*_{[\mu\nu\rho\lambda}H^*_{\sigma]} + C^*_{[\mu\nu\rho}C^*_{\lambda\sigma]} + C^*_{[\mu\nu\rho}H^*_\lambda H^*_{\sigma]} + \\
&+ C^*_{[\mu\nu}C^*_{\rho\lambda}H^*_{\sigma]} + C^*_{[\mu\nu}H^*_\rho H^*_\lambda H^*_{\sigma]} + \\
&+ H^*_\mu H^*_\nu H^*_\rho H^*_\lambda H^*_\sigma\Big)\mathcal{G}^{\mu\nu\rho\lambda\sigma}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon}\,.
\end{aligned} \tag{E.37}
$$

The pieces $\left(X^{(7)}_a\right)_{a=\overline{0,5}}$ are

$$
\begin{aligned}
X^{(7)}_0 = \frac{3}{5}\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(&-\frac{1}{2}\mathcal{G}^*_{\alpha\beta\gamma\delta\varepsilon}CC + \mathcal{G}^*_{[\alpha\beta\gamma\delta}C_{\varepsilon]}C + K^*_{[\alpha\beta\gamma}\phi_{\delta\varepsilon]}C - \\
&- K^*_{[\alpha\beta\gamma}C_\delta C_{\varepsilon]} - \frac{1}{3}\phi_{[\alpha\beta}\phi_{\gamma\delta}C_{\varepsilon]}\Big)\,,
\end{aligned} \tag{E.38}
$$

$$
\begin{aligned}
X^{(7)}_1 = \frac{3}{5}\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(&-\frac{1}{2}H^*_{[\alpha}\mathcal{G}^*_{\beta\gamma\delta\varepsilon]}CC + H^*_{[\alpha}K^*_{\beta\gamma\delta}C_{\varepsilon]}C - \\
&- H^*_{[\alpha}\phi_{\beta\gamma}C_\delta C_{\varepsilon]} + \frac{1}{3}H^*_{[\alpha}\phi_{\beta\gamma}\phi_{\delta\varepsilon]}\Big)\,,
\end{aligned} \tag{E.39}
$$

$$
\begin{aligned}
X^{(7)}_2 = \frac{1}{5}\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[&-\frac{3}{2}\left(C^*_{[\alpha\beta}K^*_{\gamma\delta\varepsilon]} + H^*_{[\alpha}H^*_\beta K^*_{\gamma\delta\varepsilon]}\right)CC + \\
&+ \left(C^*_{[\alpha\beta}\phi_{\gamma\delta}C_{\varepsilon]} + H^*_{[\alpha}H^*_\beta\phi_{\gamma\delta}C_{\varepsilon]}\right)C - \\
&- \left(C^*_{[\alpha\beta}C_\gamma C_\delta C_{\varepsilon]} + H^*_{[\alpha}H^*_\beta C_\gamma C_\delta C_{\varepsilon]}\right)\Big]\,,
\end{aligned} \tag{E.40}
$$

$$
\begin{aligned}
X^{(7)}_3 = \frac{1}{5}\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big[&\left(C^*_{[\alpha\beta\gamma}C_\delta C_{\varepsilon]} + C^*_{[\alpha\beta}H^*_\gamma C_\delta C_{\varepsilon]} + H^*_{[\alpha}H^*_\beta H^*_\gamma C_\delta C_{\varepsilon]}\right) - \\
&- \frac{1}{2}\left(C^*_{[\alpha\beta\gamma}\phi_{\delta\varepsilon]} + C^*_{[\alpha\beta}H^*_\gamma\phi_{\delta\varepsilon]} + H^*_{[\alpha}H^*_\beta H^*_\gamma\phi_{\delta\varepsilon]}\right)C\Big]C\,,
\end{aligned} \tag{E.41}
$$

$$
\begin{aligned}
X^{(7)}_4 = -\frac{1}{10}\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(&C^*_{[\alpha\beta\gamma\delta}C_{\varepsilon]} + C^*_{[\alpha\beta\gamma}H^*_\delta C_{\varepsilon]} + C^*_{[\alpha\beta}C^*_{\gamma\delta}C_{\varepsilon]} + \\
&+ C^*_{[\alpha\beta}H^*_\gamma H^*_\delta C_{\varepsilon]} + H^*_{[\alpha}H^*_\beta H^*_\gamma H^*_\delta C_{\varepsilon]}\Big)CC\,,
\end{aligned} \tag{E.42}
$$

$$
\begin{aligned}
X^{(7)}_5 = \frac{1}{30}\epsilon^{\alpha\beta\gamma\delta\varepsilon}\Big(&C^*_{\alpha\beta\gamma\delta\varepsilon} + C^*_{[\alpha\beta\gamma\delta}H^*_{\varepsilon]} + C^*_{[\alpha\beta\gamma}C^*_{\delta\varepsilon]} + C^*_{[\alpha\beta\gamma}H^*_\delta H^*_{\varepsilon]} + \\
&+ C^*_{[\alpha\beta}C^*_{\gamma\delta}H^*_{\varepsilon]} + C^*_{[\alpha\beta}H^*_\gamma H^*_\delta H^*_{\varepsilon]} + H^*_\alpha H^*_\beta H^*_\gamma H^*_\delta H^*_\varepsilon\Big)CCC\,.
\end{aligned} \tag{E.43}
$$

Finally, the coefficients $\left(X^{(8)}_a\right)_{a=\overline{0,5}}$ are given by

$$
\begin{aligned}
X^{(8)}_0 = -\frac{1}{200}\Big[&60\left(C^*\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon} - \frac{2}{25}C^{*[\alpha}\mathcal{G}^{\beta\gamma\delta\varepsilon]} + \frac{1}{5}\phi^{*[\alpha\beta}K^{\gamma\delta\varepsilon]}\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} + \\
&+ \phi^{*\alpha\beta}\epsilon_{\alpha\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\beta\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'} -
\end{aligned}
$$

$$- \mathcal{G}^{\alpha\beta\gamma\delta}\epsilon_{\alpha\beta\mu\nu\rho}K^{\mu\nu\rho}\epsilon_{\gamma\delta\mu'\nu'\rho'}K^{\mu'\nu'\rho'}\Bigg] , \tag{E.44}$$

$$\begin{aligned} X_1^{(8)} &= \frac{1}{20}\epsilon_{\beta\mu\nu\rho\lambda}H^*_\alpha\Bigg(\frac{1}{10}K^{\alpha\beta\gamma}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\gamma\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'} - \\ &\qquad - \frac{1}{3}\mathcal{G}^{\alpha\beta\lambda\delta\varepsilon}K^{\mu\nu\rho}\epsilon_{\delta\varepsilon\mu'\nu'\rho'}K^{\mu'\nu'\rho'}\Bigg) + \\ &\quad + \frac{3}{5}H^*_\mu\left(-2\phi^{*\mu\alpha}\mathcal{G}^{\beta\gamma\delta\varepsilon} + \frac{1}{2}C^{*\mu}\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon}\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} , \end{aligned} \tag{E.45}$$

$$\begin{aligned} X_2^{(8)} &= -\frac{2}{(5!)^2}\epsilon^{\alpha\beta\gamma\delta\varepsilon}\left(C^*_{\alpha\beta} + H^*_\alpha H^*_\beta\right)\epsilon_{\gamma\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\delta\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'}\epsilon_{\varepsilon\mu''\nu''\rho''\lambda''}\mathcal{G}^{\mu''\nu''\rho''\lambda''} + \\ &\quad + \frac{3}{10}\left(C^*_{\mu\nu} + H^*_\mu H^*_\nu\right)\left(-2K^{\mu\nu\alpha}\mathcal{G}^{\beta\gamma\delta\varepsilon} + \phi^{*\mu\nu}\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon}\right)\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} , \end{aligned} \tag{E.46}$$

$$\begin{aligned} X_3^{(8)} &= \frac{1}{60}\left(C^*_{\alpha\beta\gamma} + C^*_{[\alpha\beta}H^*_{\gamma]} + H^*_\alpha H^*_\beta H^*_\gamma\right)\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon}\epsilon_{\delta\mu\nu\rho\lambda}\mathcal{G}^{\mu\nu\rho\lambda}\epsilon_{\varepsilon\mu'\nu'\rho'\lambda'}\mathcal{G}^{\mu'\nu'\rho'\lambda'} - \\ &\quad - \frac{1}{10}\left(C^*_{\mu\nu\rho} + C^*_{[\mu\nu}H^*_{\rho]} + H^*_\mu H^*_\nu H^*_\rho\right)K^{\mu\nu\rho}\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} , \end{aligned} \tag{E.47}$$

$$\begin{aligned} X_4^{(8)} &= -\frac{1}{20}\Big(C^*_{\mu\nu\rho\lambda} + C^*_{[\mu\nu\rho}H^*_{\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]} + C^*_{[\mu\nu}H^*_\rho H^*_{\lambda]} + \\ &\qquad + H^*_\mu H^*_\nu H^*_\rho H^*_\lambda\Big)\mathcal{G}^{\mu\nu\rho\lambda}\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} , \end{aligned} \tag{E.48}$$

$$\begin{aligned} X_5^{(8)} &= -\frac{1}{60}\Big(C^*_{\mu\nu\rho\lambda\sigma} + C^*_{[\mu\nu\rho\lambda}H^*_{\sigma]} + C^*_{[\mu\nu\rho}C^*_{\lambda\sigma]} + C^*_{[\mu\nu\rho}H^*_\lambda H^*_{\sigma]} + C^*_{[\mu\nu}C^*_{\rho\lambda}H^*_{\sigma]} + \\ &\qquad + C^*_{[\mu\nu}H^*_\rho H^*_\lambda H^*_{\sigma]} + H^*_\mu H^*_\nu H^*_\rho H^*_\lambda H^*_\sigma\Big)\mathcal{G}^{\mu\nu\rho\lambda\sigma}\mathcal{G}^{\alpha\beta\gamma\delta\varepsilon}\mathcal{G}_{\alpha\beta\gamma\delta\varepsilon} . \end{aligned} \tag{E.49}$$

F. Commutators among the deformed gauge transformations

We have seen in section 5 that the terms of antighost number one in the deformed solution to the master equation (4.46) provide the gauge transformations (5.2)–(5.7) of the interacting theory. On behalf of these gauge transformations we are able to identify the nontrivial gauge generators of all fields. In terms of the notations (2.14)–(2.18) and taking into account the values (2.20)–(2.21) of the BRST generators, we observe that the terms of antighost number one appearing in the deformed solution (4.46) may be generically written like $\Phi^*_{\alpha_0}Z^{\alpha_0}_{\ \alpha_1}\eta^{\alpha_1}$ in De Witt condensed notations, where the functions $Z^{\alpha_0}_{\ \alpha_1}$ are the precisely the gauge generators of the deformed gauge transformations. Identifying the functions $Z^{\alpha_0}_{\ \alpha_1}$ for each BF field, we initially display the concrete form of the nonvanishing, deformed gauge generators. In this way we determine the gauge generators of the one-form A^μ (written for convenience in De Witt condensed notations) like

$$(\tilde{Z}^\mu_{(A)}) = (Z^\mu_{(A)}) = \partial^\mu,\ (\tilde{Z}^\mu_{(A)})_\alpha = -2gW_2(\varphi)\,\delta^\mu_\alpha,\ (\tilde{Z}^\mu_{(A)})_{\alpha\beta\gamma\delta} = -2gW_6(\varphi)\,\sigma^{\mu\nu}\varepsilon_{\nu\alpha\beta\gamma\delta} , \tag{F.1}$$

while for the other one-form, H^μ, we can write

$$(\tilde{Z}^\mu_{(H)})_{\alpha\beta} = -D_{[\alpha}\delta^\mu_{\beta]} , \tag{F.2}$$

$$\begin{aligned}\left(\tilde{Z}^{\mu}_{(H)}\right) &= g\Bigg[\frac{dW_1}{d\varphi}(\varphi)\,H^{\mu} - 3\frac{dW_1}{d\varphi}(\varphi)\,K^{\mu\nu\rho}\phi_{\nu\rho} + \\ &\quad + \frac{1}{4}\varepsilon^{\mu\nu\rho\lambda\sigma}\left(\frac{dW_4}{d\varphi}(\varphi)\,\varepsilon_{\nu\rho\alpha\beta\gamma}K^{\alpha\beta\gamma}\varepsilon_{\lambda\sigma\alpha'\beta'\gamma'}K^{\alpha'\beta'\gamma'} + \right. \\ &\quad \left. + \frac{dW_5}{d\varphi}(\varphi)\,\phi_{\nu\rho}\phi_{\lambda\sigma}\right)\Bigg], \qquad \text{(F.3)}\end{aligned}$$

$$\left(\tilde{Z}^{\mu}_{(H)}\right)_{\alpha\beta\gamma} = -g\left(\frac{dW_2}{d\varphi}(\varphi)\,\delta^{\mu}_{[\alpha}\phi_{\beta\gamma]} + 3\frac{dW_6}{d\varphi}(\varphi)\,K^{\mu\nu\rho}\varepsilon_{\nu\rho\alpha\beta\gamma}\right), \qquad \text{(F.4)}$$

$$\begin{aligned}\left(\tilde{Z}^{\mu}_{(H)}\right)^{\alpha} &= g\Bigg[2\left(\frac{dW_2}{d\varphi}(\varphi)\,B^{\mu\alpha} + 3\frac{dW_3}{d\varphi}(\varphi)\,K^{\mu\nu\alpha}A_{\nu}\right) - \\ &\quad - \frac{dW_5}{d\varphi}(\varphi)\,\varepsilon^{\mu\nu\rho\lambda\alpha}\phi_{\nu\rho}A_{\lambda}\Bigg], \qquad \text{(F.5)}\end{aligned}$$

$$\begin{aligned}\left(\tilde{Z}^{\mu}_{(H)}\right)_{\alpha\beta\gamma\delta} &= g\Bigg(\frac{dW_3}{d\varphi}(\varphi)\,\delta^{\mu}_{[\alpha}A_{\beta}\phi_{\gamma\delta]} + 12\frac{dW_4}{d\varphi}(\varphi)\,K^{\mu\nu\rho}A_{\nu}\varepsilon_{\rho\alpha\beta\gamma\delta} + \\ &\quad + 2\frac{dW_6}{d\varphi}(\varphi)\,B^{\mu\nu}\varepsilon_{\nu\alpha\beta\gamma\delta}\Bigg). \qquad \text{(F.6)}\end{aligned}$$

There is a single nontrivial, deformed gauge generator associated with the scalar field φ, which reads as

$$(\tilde{Z}_{(\varphi)}) = -gW_1(\varphi). \qquad \text{(F.7)}$$

Along the same line, we obtain the nonvanishing gauge generators of the two-forms $B^{\mu\nu}$ and $\phi_{\mu\nu}$ of the type

$$(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma} = (Z^{\mu\nu}_{(B)})_{\alpha\beta\gamma} = -\frac{1}{2}\partial_{[\alpha}\delta^{\mu}_{\beta}\delta^{\nu}_{\gamma]}, \qquad (\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta} = -gW_1(\varphi)\,\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta]}, \qquad \text{(F.8)}$$

$$(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta} = g\left(W_3(\varphi)\,\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta}\phi_{\gamma\delta]} + 12W_4(\varphi)\,K^{\mu\nu\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\right), \qquad \text{(F.9)}$$

$$(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha} = g\left(6W_3(\varphi)\,K^{\mu\nu\alpha} - W_5(\varphi)\,\varepsilon^{\mu\nu\rho\lambda\alpha}\phi_{\rho\lambda}\right), \qquad \text{(F.10)}$$

$$(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha} = D^{(-)[\mu}\delta^{\nu]}_{\alpha}, \qquad (\tilde{Z}^{\mu\nu}_{(\phi)}) = 3g\left(W_3(\varphi)\,\phi^{\mu\nu} + 2W_4(\varphi)\,\varepsilon^{\mu\nu\rho\lambda\sigma}K_{\rho\lambda\sigma}\right), \qquad \text{(F.11)}$$

$$(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha\beta\gamma} = 3gW_6(\varphi)\,\varepsilon^{\mu\nu\alpha\beta\gamma}, \qquad (\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha\beta\gamma\delta} = -6gW_4(\varphi)\,A^{[\mu}\varepsilon^{\nu]\alpha\beta\gamma\delta}. \qquad \text{(F.12)}$$

Finally, the three-form $K^{\mu\nu\rho}$ displays the following nontrivial, deformed gauge generators:

$$(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta} = -\frac{1}{6}D^{(+)}_{[\alpha}\delta^{\mu}_{\beta}\delta^{\nu}_{\gamma}\delta^{\rho}_{\delta]}, \qquad (\tilde{Z}^{\mu\nu\rho}_{(K)})^{\alpha} = -gW_5(\varphi)\,\varepsilon^{\mu\nu\rho\lambda\alpha}A_{\lambda}, \qquad \text{(F.13)}$$

$$(\tilde{Z}^{\mu\nu\rho}_{(K)}) = g\left(-3W_3(\varphi)\,K^{\mu\nu\rho} + \frac{1}{2}\varepsilon^{\mu\nu\rho\lambda\sigma}W_5(\varphi)\,\phi_{\lambda\sigma}\right), \qquad \text{(F.14)}$$

$$(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma} = -\frac{1}{2}gW_2(\varphi)\,\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta}\delta^{\rho}_{\gamma]}. \qquad \text{(F.15)}$$

Maintaining the condensed notations introduced in the beginning of this section, we observe that the deformed solution (4.46) contains pieces of antighost number two generically written like $\left(\frac{1}{2}\eta^{*}_{\alpha_1}C^{\alpha_1}{}_{\beta_1\gamma_1} - \frac{1}{4}M^{\alpha_0\beta_0}_{\beta_1\gamma_1}\Phi^{*}_{\alpha_0}\Phi^{*}_{\beta_0}\right)\eta^{\beta_1}\eta^{\gamma_1}$. The coefficients $C^{\alpha_1}{}_{\beta_1\gamma_1}$ and $M^{\alpha_0\beta_0}_{\beta_1\gamma_1}$ represent the deformed structure functions of order one corresponding to the gauge algebra of the interacting theory. These structure functions determine the type of gauge algebra via the commutators among the new gauge transformations: $Z^{\beta_0}_{\ \alpha_1}\frac{\delta Z^{\alpha_0}_{\ \beta_1}}{\delta\Phi^{\beta_0}} - Z^{\beta_0}_{\ \beta_1}\frac{\delta Z^{\alpha_0}_{\ \alpha_1}}{\delta\Phi^{\beta_0}} = C^{\gamma_1}_{\ \ \alpha_1\beta_1}Z^{\alpha_0}_{\ \gamma_1} + M^{\alpha_0\beta_0}_{\alpha_1\beta_1}\frac{\delta\tilde{S}}{\delta\Phi^{\beta_0}}$. Thus, if at least one coefficient $M^{\alpha_0\beta_0}_{\alpha_1\beta_1}$ is nonvanishing, then the gauge algebra is open, or, in other words, only closes on-shell. In the opposite situation the gauge algebra is closed, but it may be abelian (all the functions $C^{\gamma_1}_{\ \ \alpha_1\beta_1}$ are vanishing) or non-abelian. After analyzing all the terms from the deformed solution to the master equation that contribute to the gauge algebra relations, we are able to write the expressions of all commutators corresponding to the interacting BF model. In order to keep at minimum the number of these relations we omit the abelian commutators from the list. In this manner we obtain some nonvanishing commutators involving the gauge generators of the one form A^μ like

$$(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu}_{(A)})_{\alpha}}{\delta\varphi} = -3gW_3(\tilde{Z}^{\mu}_{(A)})_{\alpha} - \frac{1}{4}gW_5\varepsilon^{\omega\theta\varphi\pi\tau}\sigma_{\tau\alpha}(\tilde{Z}^{\mu}_{(A)})_{\omega\theta\varphi\pi}\,, \tag{F.16}$$

$$(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu}_{(A)})_{\alpha\beta\gamma\delta}}{\delta\varphi} = -6gW_4\varepsilon_{\alpha\beta\gamma\delta\eta}\sigma^{\eta\omega}(\tilde{Z}^{\mu}_{(A)})_{\omega} + 3gW_3(\tilde{Z}^{\mu}_{(A)})_{\alpha\beta\gamma\delta}\,, \tag{F.17}$$

$$\begin{aligned}(\tilde{Z}^{\omega}_{(A)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})^{\varepsilon}}{\delta A^{\omega}} - (\tilde{Z}^{\omega}_{(A)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta}}{\delta A^{\omega}} &= \\ = \frac{1}{3}gW_3\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\delta^{\varepsilon}_{\delta]}(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'}\,,\end{aligned} \tag{F.18}$$

$$\begin{aligned}(\tilde{Z}^{\omega}_{(A)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\theta\varphi\pi\tau}}{\delta A^{\omega}} - (\tilde{Z}^{\omega}_{(A)})_{\theta\varphi\pi\tau}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta}}{\delta A^{\omega}} &= \\ = \frac{1}{3}gW_4\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\varepsilon_{\delta]\theta\varphi\pi\tau}(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'}\,,\end{aligned} \tag{F.19}$$

$$\begin{aligned}(\tilde{Z}^{\omega}_{(A)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})^{\beta}}{\delta A^{\omega}} - (\tilde{Z}^{\omega}_{(A)})^{\beta}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})^{\alpha}}{\delta A^{\omega}} &= \\ = \frac{1}{3}gW_5\varepsilon^{\alpha\beta\alpha'\beta'\gamma'}(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'}\,,\end{aligned} \tag{F.20}$$

$$\begin{aligned}(\tilde{Z}^{\omega}_{(A)})\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta}}{\delta A^{\omega}} + (\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta}}{\delta\varphi} - (\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})}{\delta K^{\omega\theta\pi}} - (\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})}{\delta\phi^{\omega\theta}} &= \\ = \frac{1}{8}gW_3\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\delta^{\delta'}_{\delta]}(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'\delta'} - 6gW_4\varepsilon_{\alpha\beta\gamma\delta\alpha'}(\tilde{Z}^{\mu\nu\rho}_{(K)})^{\alpha'} - \\ -\frac{1}{2}g\frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta}\delta^{\rho}_{\gamma}\delta^{\alpha'}_{\delta]}\,,\end{aligned} \tag{F.21}$$

$$\begin{aligned}(\tilde{Z}^{\omega}_{(A)})\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})^{\alpha}}{\delta A^{\omega}} + (\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})^{\alpha}}{\delta\varphi} - (\tilde{Z}^{\omega\theta\pi}_{(K)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})}{\delta K^{\omega\theta\pi}} - (\tilde{Z}^{\omega\theta}_{(\phi)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(K)})}{\delta\phi^{\omega\theta}} &= \\ = -\frac{1}{4}gW_5\varepsilon^{\alpha\alpha'\beta'\gamma'\delta'}(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'\delta'} - 3gW_3\delta^{\alpha}_{\alpha'}(\tilde{Z}^{\mu\nu\rho}_{(K)})^{\alpha'} - \\ -g\frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\varepsilon^{\mu\nu\rho\alpha\alpha'}\,,\end{aligned} \tag{F.22}$$

$$(\tilde{Z}^{\omega}_{(A)})_{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})_{\beta}}{\delta A^{\omega}}-(\tilde{Z}^{\omega}_{(A)})_{\beta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha}}{\delta A^{\omega}}= \\ =\frac{1}{3}gW_5\varepsilon_{\alpha\beta\alpha'\beta'\gamma'}(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha'\beta'\gamma'}, \tag{F.23}$$

$$(\tilde{Z}^{\omega}_{(A)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})_{\theta\varphi\pi\tau}}{\delta A^{\omega}}-(\tilde{Z}^{\omega}_{(A)})_{\theta\varphi\pi\tau}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha\beta\gamma\delta}}{\delta A^{\omega}}= \\ =\frac{1}{3}gW_4\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\varepsilon_{\delta]\theta\varphi\pi\tau}(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'\beta'\gamma'}, \tag{F.24}$$

$$(\tilde{Z}^{\omega}_{(A)})^{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})_{\varepsilon}}{\delta A^{\omega}}-(\tilde{Z}^{\omega}_{(A)})_{\varepsilon}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha\beta\gamma\delta}}{\delta A^{\omega}}= \\ =\frac{1}{3}gW_3\delta^{[\alpha}_{\alpha'}\delta^{\beta}_{\beta'}\delta^{\gamma}_{\gamma'}\delta^{\delta]}_{\varepsilon}(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha'\beta'\gamma'}, \tag{F.25}$$

$$(\tilde{Z}^{\omega}_{(A)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha}}{\delta A^{\omega}}+(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha}}{\delta\varphi}-(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(\phi)})}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu\rho}_{(\phi)})}{\delta\phi^{\omega\theta}}= \\ =-3gW_3\delta^{\alpha'}_{\alpha}(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'}-\frac{1}{4}gW_5\varepsilon_{\alpha\alpha'\beta'\gamma'\delta'}(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha'\beta'\gamma'\delta'}+ \\ +3g\frac{\delta W_3}{\delta\varphi}\left(\frac{\delta\tilde{S}}{\delta H_{\mu}}\delta^{\nu}_{\alpha}-\frac{\delta\tilde{S}}{\delta H_{\nu}}\delta^{\mu}_{\alpha}\right), \tag{F.26}$$

$$(\tilde{Z}^{\omega}_{(A)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha\beta\gamma\delta}}{\delta A^{\omega}}+(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha\beta\gamma\delta}}{\delta\varphi}-(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})^{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(\phi)})}{\delta\phi^{\omega\theta}}= \\ =-6gW_4\varepsilon^{\alpha\beta\gamma\delta\alpha'}(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'}+\frac{1}{8}gW_3\delta^{[\alpha}_{\alpha'}\delta^{\beta}_{\beta'}\delta^{\gamma}_{\gamma'}\delta^{\delta]}_{\delta'}(\tilde{Z}^{\mu\nu}_{(\phi)})^{\alpha'\beta'\gamma'\delta'}+ \\ +6g\frac{\delta W_4}{\delta\varphi}\left(\frac{\delta\tilde{S}}{\delta H_{\mu}}\varepsilon^{\nu\alpha\beta\gamma\delta}-\frac{\delta\tilde{S}}{\delta H_{\nu}}\varepsilon^{\mu\alpha\beta\gamma\delta}\right). \tag{F.27}$$

Other nonvanishing commutators implying the gauge generators of the fields φ and $K_{\mu\nu\rho}$ are given by

$$(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha}}{\delta\varphi}+(\tilde{Z}^{\omega\theta\pi}_{(K)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha}}{\delta\phi^{\omega\theta}}= \\ =-3gW_3\delta^{\alpha}_{\alpha'}(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha'}+3g\frac{dW_3}{d\varphi}K^{\alpha\alpha'\beta'}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}- \\ -\frac{1}{4}gW_5\varepsilon^{\alpha\alpha'\beta'\gamma'\delta'}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'\delta'}-\frac{1}{2}g\frac{dW_5}{d\varphi}\varepsilon^{\alpha\alpha'\beta'\gamma'\delta'}\phi_{\alpha'\beta'}(\tilde{Z}^{\mu\nu}_{(B)})_{\gamma'\delta'}, \tag{F.28}$$

$$(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta}}{\delta\varphi}=-\frac{1}{2}gW_1\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta]}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}, \tag{F.29}$$

$$(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\varphi\eta\varsigma\tau}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\varphi\eta\varsigma\tau}}{\delta\phi^{\omega\theta}}- \\ -(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\varphi\eta\varsigma\tau}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})_{\varphi\eta\varsigma\tau}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}= \\ =\frac{1}{3}gW_4\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\varepsilon_{\delta]\varphi\eta\varsigma\tau}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'}+\frac{1}{2}g\frac{dW_4}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}A_{\gamma}\varepsilon_{\delta]\varphi\eta\varsigma\tau}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}+$$

$$
+g\frac{dW_4}{d\varphi}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta}\delta^{\alpha'}_{\gamma}\varepsilon_{\delta]\varphi\eta\varsigma\tau}\,, \tag{F.30}
$$

$$
\begin{aligned}
&(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta\varphi}+(\tilde{Z}^{\omega\theta\pi}_{(K)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}=\\
&\quad=\frac{1}{8}gW_3\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\delta^{\delta'}_{\delta]}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'\delta'}+\frac{1}{2}g\frac{dW_3}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\phi_{\gamma\delta]}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}-\\
&\quad-6gW_4\varepsilon_{\alpha\beta\gamma\delta\alpha'}(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha'}+6g\frac{dW_4}{d\varphi}\varepsilon_{\alpha\beta\gamma\delta\alpha'}K^{\alpha'\beta'\gamma'}(\tilde{Z}^{\mu\nu}_{(B)})_{\beta'\gamma'}\,,
\end{aligned} \tag{F.31}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\delta}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\delta}}{\delta\phi^{\omega\theta}}=\\
&\quad=\frac{1}{2}g\frac{dW_2}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\delta}_{\gamma]}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}\,,
\end{aligned} \tag{F.32}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\varepsilon}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\varepsilon}}{\delta\phi^{\omega\theta}}-(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}=\\
&\quad=\frac{1}{3}gW_3\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\delta^{\varepsilon}_{\delta]}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'}-\frac{1}{2}g\frac{dW_3}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\varepsilon}_{\gamma}A_{\delta]}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}-\\
&\quad-g\frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta}\delta^{\varepsilon}_{\gamma}\delta^{\alpha'}_{\delta]}\,,
\end{aligned} \tag{F.33}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\varphi\eta\varsigma}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\varphi\eta\varsigma}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}=\\
&\quad=\frac{1}{2}g\frac{dW_6}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\varepsilon_{\gamma\delta]\varphi\eta\varsigma}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}\,,
\end{aligned} \tag{F.34}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\beta}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\beta}}{\delta\phi^{\omega\theta}}-(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\beta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})^{\beta}\frac{\delta(\tilde{Z}^{\mu\nu}_{(B)})^{\alpha}}{\delta\phi^{\omega\theta}}=\\
&\quad=\frac{1}{3}gW_5\varepsilon^{\alpha\beta\alpha'\beta'\gamma'}(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'}+\frac{1}{2}g\frac{dW_5}{d\varphi}\varepsilon^{\alpha\beta\alpha'\beta'\gamma'}A_{\alpha'}(\tilde{Z}^{\mu\nu}_{(B)})_{\beta'\gamma'}+\\
&\quad+g\frac{dW_5}{d\varphi}\varepsilon^{\mu\nu\alpha\beta\alpha'}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\,.
\end{aligned} \tag{F.35}
$$

Finally, the remaining nonvanishing commutators that involve the gauge generators of the field H^{μ} are expressed like

$$
\begin{aligned}
&(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta\varphi}+(\tilde{Z}^{\omega}_{(A)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta A^{\omega}}+(\tilde{Z}^{\omega\theta\pi}_{(K)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta K^{\omega\theta\pi}}+\\
&+(\tilde{Z}^{\omega\theta}_{(\phi)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta\phi^{\omega\theta}}-(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu}_{(H)})}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu}_{(H)})}{\delta\phi^{\omega\theta}}-(\tilde{Z}^{\omega}_{(H)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu}_{(H)})}{\delta H^{\omega}}=\\
&=-3gW_3\delta^{\alpha}_{\alpha'}(\tilde{Z}^{\mu}_{(H)})^{\alpha'}+3g\frac{dW_3}{d\varphi}K^{\alpha\alpha'\beta'}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}-6g\frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta\phi_{\mu\alpha}}-\\
&-\frac{1}{4}gW_5\varepsilon^{\alpha\alpha'\beta'\gamma'\delta'}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'\gamma'\delta'}-\frac{1}{2}g\frac{dW_5}{d\varphi}\varepsilon^{\alpha\alpha'\beta'\gamma'\delta'}\phi_{\alpha'\beta'}(\tilde{Z}^{\mu}_{(H)})_{\gamma'\delta'}+\\
&+6g\frac{d^2W_3}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}K^{\mu\alpha\alpha'}-g\frac{dW_5}{d\varphi}\frac{\delta\tilde{S}}{\delta K^{\alpha'\beta'\gamma'}}\varepsilon^{\mu\alpha\alpha'\beta'\gamma'}-
\end{aligned}
$$

$$-g\frac{d^2W_5}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\varepsilon^{\mu\alpha\alpha'\beta'\gamma'}\phi_{\beta'\gamma'}\,, \tag{F.36}$$

$$\begin{aligned}
&(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta\varphi}+(\tilde{Z}^{\omega}_{(A)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta A^{\omega}}+\\
&+(\tilde{Z}^{\omega\theta\pi}_{(K)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}-\\
&-(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})}{\delta\phi^{\omega\theta}}-(\tilde{Z}^{\omega}_{(H)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})}{\delta H^{\omega}}=\\
&=\frac{1}{8}gW_3\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\delta^{\delta'}_{\delta]}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'\gamma'\delta'}+\frac{1}{2}g\frac{dW_3}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\phi_{\gamma\delta]}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}-\\
&-6gW_4\varepsilon_{\alpha\beta\gamma\delta\alpha'}(\tilde{Z}^{\mu}_{(H)})^{\alpha'}+6g\frac{dW_4}{d\varphi}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}K^{\varepsilon\alpha'\beta'}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}-\\
&-g\frac{d^2W_3}{d\varphi^2}\delta^{\mu}_{[\alpha}\delta^{\varepsilon}_{\beta}\phi_{\gamma\delta]}\frac{\delta\tilde{S}}{\delta H^{\varepsilon}}-\frac{1}{2}g\frac{dW_3}{d\varphi}\delta^{\mu}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\delta^{\delta'}_{\delta]}\frac{\delta\tilde{S}}{\delta K^{\beta'\gamma'\delta'}}-\\
&-12g\frac{dW_4}{d\varphi}\frac{\delta\tilde{S}}{\delta\phi_{\mu\varepsilon}}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}-12g\frac{d^2W_4}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}K^{\mu\alpha'\beta'}\varepsilon_{\alpha\beta\gamma\delta\beta'}\,,
\end{aligned}\tag{F.37}$$

$$\begin{aligned}
&(\tilde{Z}^{\omega}_{(A)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\beta}}{\delta A^{\omega}}+(\tilde{Z}^{\omega\theta}_{(B)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\beta}}{\delta B^{\omega\theta}}+(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\beta}}{\delta K^{\omega\theta\pi}}+\\
&+(\tilde{Z}^{\omega\theta}_{(\phi)})^{\alpha}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\beta}}{\delta\phi^{\omega\theta}}-(\tilde{Z}^{\omega}_{(A)})^{\beta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta A^{\omega}}-(\tilde{Z}^{\omega\theta}_{(B)})^{\beta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta B^{\omega\theta}}-\\
&-(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\beta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})^{\beta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\alpha}}{\delta\phi^{\omega\theta}}=\\
&=\frac{1}{3}gW_5\varepsilon^{\alpha\beta\alpha'\beta'\gamma'}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'\gamma'}+\frac{1}{2}g\frac{dW_5}{d\varphi}\varepsilon^{\alpha\beta\gamma\alpha'\beta'}A_{\gamma}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}-\\
&-g\frac{dW_5}{d\varphi}\frac{\delta\tilde{S}}{\delta B^{\alpha'\beta'}}\varepsilon^{\mu\alpha\beta\alpha'\beta'}-g\frac{d^2W_5}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}A_{\beta'}\varepsilon^{\mu\alpha\beta\alpha'\beta'}\,,
\end{aligned}\tag{F.38}$$

$$\begin{aligned}
&(\tilde{Z}^{\omega}_{(A)})_{\alpha\beta\gamma\delta}\tfrac{\delta(\tilde{Z}^{\mu}_{(H)})^{\varepsilon}}{\delta A^{\omega}}+(\tilde{Z}^{\omega\theta}_{(B)})_{\alpha\beta\gamma\delta}\tfrac{\delta(\tilde{Z}^{\mu}_{(H)})^{\varepsilon}}{\delta B^{\omega\theta}}+\\
&+(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\varepsilon}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\varepsilon}}{\delta\phi^{\omega\theta}}-(\tilde{Z}^{\omega}_{(A)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta A^{\omega}}-\\
&-(\tilde{Z}^{\omega\theta}_{(B)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta B^{\omega\theta}}-(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}=\\
&=\frac{1}{3}gW_3\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\delta^{\varepsilon}_{\delta]}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'\gamma'}-\frac{1}{2}g\frac{dW_3}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\varepsilon}_{\gamma}A_{\delta]}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}-\\
&-g\frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta B^{\alpha'\beta'}}\delta^{\alpha}_{[\alpha}\delta^{\varepsilon}_{\beta}\delta^{\alpha'}_{\gamma}\delta^{\beta'}_{\delta]}-g\frac{d^2W_3}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}A_{\beta'}\delta^{\alpha}_{[\alpha}\delta^{\varepsilon}_{\beta}\delta^{\alpha'}_{\gamma}\delta^{\beta'}_{\delta]}\,,
\end{aligned}\tag{F.39}$$

$$\begin{aligned}
&(\tilde{Z}_{(\varphi)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta}}{\delta\varphi}+(\tilde{Z}^{\omega}_{(A)})\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta}}{\delta A^{\omega}}-(\tilde{Z}^{\omega}_{(H)})_{\alpha\beta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})}{\delta H^{\omega}}=\\
&=-\frac{1}{2}g\frac{dW_1}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta]}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}+g\frac{d^2W_1}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\delta^{\mu}_{[\alpha}\delta^{\alpha'}_{\beta]}\,,
\end{aligned}\tag{F.40}$$

$$(\tilde{Z}^{\omega\theta}_{(B)})_{\alpha\beta\gamma}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\varepsilon}}{\delta B^{\omega\theta}}+(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\varepsilon}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma}\frac{\delta(\tilde{Z}^{\mu}_{(H)})^{\varepsilon}}{\delta \phi^{\omega\theta}}-$$
$$-(\tilde{Z}^{\omega\theta\pi}_{(K)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})^{\varepsilon}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma}}{\delta\phi^{\omega\theta}}=$$
$$=\frac{1}{2}g\frac{dW_2}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\varepsilon}_{\gamma]}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}+g\frac{d^2W_2}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\delta^{\mu}_{[\alpha}\delta^{\varepsilon}_{\beta}\delta^{\alpha'}_{\gamma]}\,, \tag{F.41}$$

$$(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\varphi\eta\zeta}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\varphi\eta\zeta}}{\delta\phi^{\omega\theta}}-$$
$$-(\tilde{Z}^{\omega\theta}_{(B)})_{\varphi\eta\zeta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta B^{\omega\theta}}-(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\varphi\eta\zeta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})_{\varphi\eta\zeta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}=$$
$$=-g\frac{dW_6}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\varepsilon_{\gamma\delta]\varphi\eta\zeta}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}+g\frac{d^2W_6}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}\delta^{\mu}_{[\varphi}\delta^{\alpha'}_{\eta}\varepsilon_{\zeta]\alpha\beta\gamma\delta}\,, \tag{F.42}$$

$$(\tilde{Z}^{\omega}_{(A)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\varphi\eta\zeta\tau}}{\delta A^{\omega}}+(\tilde{Z}^{\omega\theta}_{(B)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\varphi\eta\zeta\tau}}{\delta B^{\omega\theta}}+$$
$$+(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\varphi\eta\zeta\tau}}{\delta K^{\omega\theta\pi}}+(\tilde{Z}^{\omega\theta}_{(\phi)})_{\alpha\beta\gamma\delta}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\varphi\eta\zeta\tau}}{\delta\phi^{\omega\theta}}-$$
$$-(\tilde{Z}^{\omega}_{(A)})_{\varphi\eta\zeta\tau}\frac{\delta\left(\tilde{Z}^{\mu}_{(H)}\right)_{\alpha\beta\gamma\delta}}{\delta A^{\omega}}-(\tilde{Z}^{\omega\theta}_{(B)})_{\varphi\eta\zeta\tau}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta B^{\omega\theta}}-$$
$$-(\tilde{Z}^{\omega\theta\pi}_{(K)})_{\varphi\eta\zeta\tau}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta K^{\omega\theta\pi}}-(\tilde{Z}^{\omega\theta}_{(\phi)})_{\varphi\eta\zeta\tau}\frac{\delta(\tilde{Z}^{\mu}_{(H)})_{\alpha\beta\gamma\delta}}{\delta\phi^{\omega\theta}}=$$
$$=\frac{1}{3}gW_4\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}\delta^{\gamma'}_{\gamma}\varepsilon_{\delta]\varphi\eta\zeta\tau}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'\gamma'}+\frac{1}{2}g\frac{dW_4}{d\varphi}\delta^{\alpha'}_{[\alpha}\delta^{\beta'}_{\beta}A_{\gamma}\varepsilon_{\delta]\varphi\eta\zeta\tau}(\tilde{Z}^{\mu}_{(H)})_{\alpha'\beta'}-$$
$$-2g\frac{dW_4}{d\varphi}\frac{\delta\tilde{S}}{\delta B^{\alpha'\beta'}}\delta^{\mu}_{[\alpha}\delta^{\alpha'}_{\beta}\delta^{\beta'}_{\gamma}\varepsilon_{\delta]\varphi\eta\zeta\tau}-2g\frac{d^2W_4}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^{\alpha'}}A_{\beta'}\delta^{\mu}_{[\alpha}\delta^{\alpha'}_{\beta}\delta^{\beta'}_{\gamma}\varepsilon_{\delta]\varphi\eta\zeta\tau}\,. \tag{F.43}$$

G. Reducibility of the interacting model

In what follows we focus on the reducibility functions and relations corresponding to the resulting interacting BF model. In view of this we maintain the condensed notations introduced in the appendix F and observe that the deformed solution (4.46) contains pieces of antighost numbers $k=2,3,4$, generically written like

$$\sum_{k=2}^{4}\left(c_k\eta^{*}_{\alpha_{k-2}}\Phi^{*}_{\beta_0}C^{\alpha_{k-2}\beta_0}_{\alpha_k}+\eta^{*}_{\alpha_{k-1}}Z^{\alpha_{k-1}}_{\ \ \alpha_k}\right)\eta^{\alpha_k},$$

where $c_2=\frac{1}{2}$, $c_3=-1$, and $c_4=1$. The functions $\left(Z^{\alpha_{k-1}}_{\ \ \alpha_k}\right)_{k=2,3,4}$ represent the deformed reducibility functions of order $(k-1)$ and the above terms produce the reducibility relations $Z^{\alpha_{k-2}}_{\ \ \alpha_{k-1}}Z^{\alpha_{k-1}}_{\ \ \alpha_k}=C^{\alpha_{k-2}\beta_0}_{\alpha_k}\frac{\delta\tilde{S}}{\delta\Phi^{\beta_0}}$ of order $(k-1)$, with $k=2,3,4$, associated with the interacting BF theory. If at least one coefficient $C^{\alpha_{k-2}\beta_0}_{\alpha_k}$ is nonvanishing, we say that the reducibility relations of order $(k-1)$ take place on-shell, while in the opposite situation we say that the $(k-1)$-level reducibility holds off-shell. In order to keep the number of relations at minimum we completely omit off-shell reducibility relations.

Analyzing these kinds of terms, we obtain that the deformed first-order reducibility functions ($k=2$) read as

$$(\tilde{Z}_1) = 2gW_2(\varphi), \quad (\tilde{Z}_1)_{\mu\nu\rho\lambda\sigma} = -2g\varepsilon_{\mu\nu\rho\lambda\sigma}W_6(\varphi), \quad (\tilde{Z}_1^{\mu}) = D^{(-)\mu}, \tag{G.1}$$

$$(\tilde{Z}_1^{\mu})_{\alpha\beta\gamma\delta\varepsilon} = 6g\varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_4(\varphi)A^{\mu}, \quad (\tilde{Z}_1^{\mu})^{\nu\rho\lambda\sigma} = -3g\varepsilon^{\mu\nu\rho\lambda\sigma}W_6(\varphi), \tag{G.2}$$

$$(\tilde{Z}_1^{\mu\nu})_{\alpha\beta\gamma} = -\frac{1}{2}D_{[\alpha}\delta_{\beta}^{\mu}\delta_{\gamma]}^{\nu}, \quad (\tilde{Z}_1^{\mu\nu\rho}) = 2gW_3(\varphi) - \frac{1}{3}gW_5(\varphi)\varepsilon^{\mu\nu\rho\lambda\sigma}\phi_{\lambda\sigma}, \tag{G.3}$$

$$\begin{aligned}(\tilde{Z}_1^{\mu\nu}) = &-g\bigg(\frac{dW_2}{d\varphi}(\varphi)B^{\mu\nu} - \\ &- 3\left(\frac{dW_3}{d\varphi}(\varphi)K^{\mu\nu\rho} - \frac{1}{6}\frac{dW_5}{d\varphi}(\varphi)\varepsilon^{\mu\nu\rho\lambda\sigma}\phi_{\lambda\sigma}\right)A_{\rho}\bigg),\end{aligned} \tag{G.4}$$

$$(\tilde{Z}_1^{\mu\nu})_{\alpha\beta\gamma\delta} = \frac{1}{2}g\left(\frac{dW_2}{d\varphi}(\varphi)\delta_{[\alpha}^{\mu}\delta_{\beta}^{\nu}\phi_{\gamma\delta]} + 6\frac{dW_3}{d\varphi}(\varphi)K^{\mu\nu\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\right), \tag{G.5}$$

$$\begin{aligned}(\tilde{Z}_1^{\mu\nu})_{\alpha\beta\gamma\delta\varepsilon} = &-\frac{1}{2}g\bigg(\frac{dW_3}{d\varphi}(\varphi)\delta_{[\alpha}^{\mu}\delta_{\beta}^{\nu}A_{\gamma}\phi_{\delta\varepsilon]} + 2\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\times \\ &\times\left(6\frac{dW_4}{d\varphi}(\varphi)K^{\mu\nu\rho}A_{\rho} - \frac{dW_6}{d\varphi}(\varphi)B^{\mu\nu}\right)\bigg),\end{aligned} \tag{G.6}$$

$$(\tilde{Z}_1^{\mu\nu\rho})_{\alpha\beta\gamma} = \frac{1}{3}gW_1(\varphi)\delta_{[\alpha}^{\mu}\delta_{\beta}^{\nu}\delta_{\gamma]}^{\rho}, \quad (\tilde{Z}_1^{\mu\nu\rho})_{\alpha\beta\gamma\delta} = (Z_1^{\mu\nu\rho})_{\alpha\beta\gamma\delta}, \tag{G.7}$$

$$(\tilde{Z}_1^{\mu\nu\rho})_{\alpha\beta\gamma\delta\varepsilon} = -4gW_4(\varphi)K^{\mu\nu\rho}\varepsilon_{\alpha\beta\gamma\delta\varepsilon} - \frac{1}{3}gW_3(\varphi)\delta_{[\alpha}^{\mu}\delta_{\beta}^{\nu}\delta_{\gamma}^{\rho}\phi_{\delta\varepsilon]}, \tag{G.8}$$

$$(\tilde{Z}_1^{\mu\nu\rho\lambda}) = -\frac{1}{4}gW_5(\varphi)\varepsilon^{\mu\nu\rho\lambda\sigma}A_{\sigma}, \quad (\tilde{Z}_1^{\mu\nu\rho\lambda})_{\alpha\beta\gamma\delta} = \frac{1}{8}gW_2(\varphi)\delta_{[\alpha}^{\mu}\delta_{\beta}^{\nu}\delta_{\gamma}^{\rho}\delta_{\delta]}^{\lambda}, \tag{G.9}$$

$$(\tilde{Z}_1^{\mu\nu\rho\lambda})_{\alpha\beta\gamma\delta\varepsilon} = -\frac{1}{24}D_{[\alpha}^{(+)}\delta_{\beta}^{\mu}\delta_{\gamma}^{\nu}\delta_{\delta}^{\rho}\delta_{\varepsilon]}^{\lambda}, \tag{G.10}$$

while the on-shell, first-order reducibility relations are given by

$$\begin{aligned}&(\tilde{Z}_{(A)}^{\mu})(\tilde{Z}_1) + (\tilde{Z}_{(A)}^{\mu})_{\alpha}(\tilde{Z}_1^{\alpha}) + (\tilde{Z}_{(A)}^{\mu})_{\alpha\beta\gamma\delta}(\tilde{Z}_1^{\alpha\beta\gamma\delta}) = \\ &\quad = 2g\frac{dW_2}{d\varphi}\frac{\delta\tilde{S}}{\delta H_{\mu}},\end{aligned} \tag{G.11}$$

$$\begin{aligned}&(\tilde{Z}_{(A)}^{\mu})(\tilde{Z}_1)_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}_{(A)}^{\mu})_{\mu'}(\tilde{Z}_1^{\mu'})_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}_{(A)}^{\mu})_{\mu'\nu'\rho'\lambda'}(\tilde{Z}_1^{\mu'\nu'\rho'\lambda'})_{\alpha\beta\gamma\delta\varepsilon} = \\ &\quad = -2g\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\frac{dW_6}{d\varphi}\frac{\delta\tilde{S}}{\delta H_{\mu}},\end{aligned} \tag{G.12}$$

$$\begin{aligned}&(\tilde{Z}_{(B)}^{\mu\nu})_{\alpha}(\tilde{Z}_1^{\alpha}) + (\tilde{Z}_{(B)}^{\mu\nu})_{\alpha\beta}(\tilde{Z}_1^{\alpha\beta}) + \\ &+(\tilde{Z}_{(B)}^{\mu\nu})_{\alpha\beta\gamma}(\tilde{Z}_1^{\alpha\beta\gamma}) + (\tilde{Z}_{(B)}^{\mu\nu})_{\alpha\beta\gamma\delta}(\tilde{Z}_1^{\alpha\beta\gamma\delta}) = \\ &\quad = 6gW_3\frac{\delta\tilde{S}}{\delta\phi_{\mu\nu}} + g\varepsilon^{\mu\nu\rho\lambda\sigma}W_5\frac{\delta\tilde{S}}{\delta K^{\rho\lambda\sigma}} + \\ &\quad\quad +g\frac{\delta\tilde{S}}{\delta H^{\rho}}\left(-6\frac{dW_3}{d\varphi}K^{\mu\nu\rho} + \frac{dW_5}{d\varphi}\varepsilon^{\mu\nu\rho\lambda\sigma}\phi_{\lambda\sigma}\right),\end{aligned} \tag{G.13}$$

$$
\begin{aligned}
&(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}(\tilde{Z}_1^{\alpha'\beta'})_{\alpha\beta\gamma} + (\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma} = \\
&\qquad = -g\frac{dW_1}{d\varphi}\frac{\delta\tilde{S}}{\delta H_\rho}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_{\gamma]}\,,
\end{aligned}
\tag{G.14}
$$

$$
\begin{aligned}
&\qquad (\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'}(\tilde{Z}_1^{\alpha'})_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'}(\tilde{Z}_1^{\alpha'\beta'})_{\alpha\beta\gamma\delta\varepsilon} + \\
&+(\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}^{\mu\nu}_{(B)})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_1^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon} = \\
&\qquad = \frac{1}{2}g\left(W_3\frac{\delta\tilde{S}}{\delta K^{\rho\lambda\sigma}} + \frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\rho}\phi_{\lambda\sigma}\right)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]} - \\
&\qquad -12g\left(W_4\frac{\delta\tilde{S}}{\delta\phi_{\mu\nu}} - \frac{dW_4}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\rho}K^{\mu\nu\rho}\right)\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\,,
\end{aligned}
\tag{G.15}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\mu\nu}_{(\phi)})(\tilde{Z}_1) + (\tilde{Z}^{\mu\nu}_{(\phi)})_\alpha(\tilde{Z}_1^\alpha) + (\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha\beta\gamma}(\tilde{Z}_1^{\alpha\beta\gamma}) + (\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha\beta\gamma\delta}(\tilde{Z}_1^{\alpha\beta\gamma\delta}) = \\
&\qquad = -6g\left(W_3\frac{\delta\tilde{S}}{\delta B_{\mu\nu}} + \frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta H_\alpha}A^\beta\delta^{[\mu}_\alpha\delta^{\nu]}_\beta\right),
\end{aligned}
\tag{G.16}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta} + (\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_1^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta} + (\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'}(\tilde{Z}_1^{\alpha'})_{\alpha\beta\gamma\delta} = \\
&\qquad = -3g\frac{dW_6}{d\varphi}\frac{\delta\tilde{S}}{\delta H_\lambda}\delta^{[\mu}_\lambda\sigma^{\nu]\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\,,
\end{aligned}
\tag{G.17}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'}(\tilde{Z}_1^{\alpha'})_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta\varepsilon} + \\
&+(\tilde{Z}^{\mu\nu}_{(\phi)})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_1^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}^{\mu\nu}_{(\phi)})(\tilde{Z}_1)_{\alpha\beta\gamma\delta\varepsilon} = \\
&\qquad = 12g\left(W_4\frac{\delta\tilde{S}}{\delta B_{\mu\nu}} + \frac{dW_4}{d\varphi}\frac{\delta\tilde{S}}{\delta H_\rho}A^\lambda\delta^{[\mu}_\rho\delta^{\nu]}_\lambda\right)\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\,,
\end{aligned}
\tag{G.18}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\mu\nu\rho}_{(K)})(\tilde{Z}_1) + (\tilde{Z}^{\mu\nu\rho}_{(K)})_\alpha(\tilde{Z}_1^\alpha) + (\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma}(\tilde{Z}_1^{\alpha\beta\gamma}) + (\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha\beta\gamma\delta}(\tilde{Z}_1^{\alpha\beta\gamma\delta}) = \\
&\qquad = -g\left(W_5\frac{\delta\tilde{S}}{\delta B^{\lambda\sigma}} + \frac{dW_5}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\lambda}A_\sigma\right)\varepsilon^{\mu\nu\rho\lambda\sigma}\,,
\end{aligned}
\tag{G.19}
$$

$$
\begin{aligned}
&(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta} + (\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_1^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta} + (\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'}(\tilde{Z}_1^{\alpha'})_{\alpha\beta\gamma\delta} = \\
&\qquad = \frac{1}{2}g\frac{dW_2}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\lambda}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_{\delta]}\,,
\end{aligned}
\tag{G.20}
$$

$$
\begin{aligned}
&\quad (\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'}(\tilde{Z}_1^{\alpha'})_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta\varepsilon} + \\
&+(\tilde{Z}^{\mu\nu\rho}_{(K)})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_1^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon} + (\tilde{Z}^{\mu\nu\rho}_{(K)})(\tilde{Z}_1)_{\alpha\beta\gamma\delta\varepsilon} = \\
&\qquad = -\frac{1}{2}g\left(W_3\frac{\delta\tilde{S}}{\delta B^{\lambda\sigma}} + \frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\lambda}A_\sigma\right)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\,,
\end{aligned}
\tag{G.21}
$$

$$
\begin{aligned}
&(\tilde{Z}^\mu_{(H)})(\tilde{Z}_1) + (\tilde{Z}^\mu_{(H)})_\alpha(\tilde{Z}_1^\alpha) + (\tilde{Z}^\mu_{(H)})_{\alpha\beta}(\tilde{Z}_1^{\alpha\beta}) + (\tilde{Z}^\mu_{(H)})_{\alpha\beta\gamma}(\tilde{Z}_1^{\alpha\beta\gamma}) + (\tilde{Z}^\mu_{(H)})_{\alpha\beta\gamma\delta}(\tilde{Z}_1^{\alpha\beta\gamma\delta}) = \\
&\qquad = -2g\frac{dW_2}{d\varphi}\frac{\delta\tilde{S}}{\delta A_\mu} - g\frac{\delta\tilde{S}}{\delta H^\nu}\left(2\frac{d^2W_2}{d\varphi^2}B^{\mu\nu} - 6\frac{d^2W_3}{d\varphi^2}K^{\mu\nu\rho}A_\rho + \frac{d^2W_5}{d\varphi^2}\varepsilon^{\mu\nu\rho\lambda\sigma}A_\rho\phi_{\lambda\sigma}\right) +
\end{aligned}
$$

$$+g\frac{\delta\tilde{S}}{\delta B^{\nu\rho}}\left(6\frac{dW_3}{d\varphi}K^{\mu\nu\rho}-\frac{dW_5}{d\varphi}\varepsilon^{\mu\nu\rho\lambda\sigma}\phi_{\lambda\sigma}\right)+$$
$$+g\frac{dW_5}{d\varphi}\frac{\delta\tilde{S}}{\delta K^{\rho\lambda\sigma}}\varepsilon^{\mu\nu\rho\lambda\sigma}A_\nu+6g\frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta\phi_{\mu\nu}}A_\nu\,, \tag{G.22}$$

$$(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'}(\tilde{Z}_1^{\alpha'\beta'})_{\alpha\beta\gamma}+(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma}=$$
$$=g\left(\frac{dW_1}{d\varphi}\frac{\delta\tilde{S}}{\delta B^{\nu\rho}}+\frac{d^2W_1}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^\nu}A_\rho\right)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_{\gamma]}\,, \tag{G.23}$$

$$(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'}(\tilde{Z}_1^{\alpha'\beta'})_{\alpha\beta\gamma\delta}+(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta}+$$
$$+(\tilde{Z}^\mu_{(H)})_{\alpha'}(\tilde{Z}_1^{\alpha'})_{\alpha\beta\gamma\delta}+(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_1^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta}=$$
$$=g\frac{\delta\tilde{S}}{\delta H^\nu}\left(\frac{1}{2}\frac{d^2W_2}{d\varphi^2}\phi_{\rho\lambda}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_{\delta]}+6\frac{d^2W_6}{d\varphi^2}K^{\mu\nu\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\right)+$$
$$+\frac{1}{2}g\frac{dW_2}{d\varphi}\frac{\delta\tilde{S}}{\delta K^{\nu\rho\lambda}}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_{\delta]}+6\frac{\delta W_6}{\delta\varphi}\frac{\delta\tilde{S}}{\delta\phi_{\mu\nu}}\varepsilon_{\nu\alpha\beta\gamma\delta}\,, \tag{G.24}$$

$$(\tilde{Z}^\mu_{(H)})(\tilde{Z}_1)_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}^\mu_{(H)})_{\alpha'}(\tilde{Z}_1^{\alpha'})_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'}(\tilde{Z}_1^{\alpha'\beta'})_{\alpha\beta\gamma\delta\varepsilon}+$$
$$+(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'\gamma'}(\tilde{Z}_1^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}^\mu_{(H)})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_1^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon}=$$
$$=g\frac{\delta\tilde{S}}{\delta H^\nu}\left(2\left(\frac{d^2W_6}{d\varphi^2}B^{\mu\nu}-6\frac{d^2W_4}{d\varphi^2}K^{\mu\nu\rho}A_\rho\right)\varepsilon_{\alpha\beta\gamma\delta\varepsilon}-\right.$$
$$\left.-\frac{1}{2}\frac{d^2W_3}{d\varphi^2}A_\rho\phi_{\lambda\sigma}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\right)+2g\frac{dW_6}{d\varphi}\frac{\delta\tilde{S}}{\delta A_\mu}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}-$$
$$-g\frac{\delta\tilde{S}}{\delta B^{\nu\rho}}\left(12\frac{dW_4}{d\varphi}K^{\mu\nu\rho}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}+\frac{1}{2}\frac{dW_3}{d\varphi}\phi_{\lambda\sigma}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\right)-$$
$$-12g\frac{dW_4}{d\varphi}\frac{\delta\tilde{S}}{\delta\phi_{\mu\nu}}A_\nu\varepsilon_{\alpha\beta\gamma\delta\varepsilon}+\frac{1}{2}g\frac{dW_3}{d\varphi}\frac{\delta\tilde{S}}{\delta K^{\rho\lambda\sigma}}A_\nu\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\,. \tag{G.25}$$

In the next step we determine the deformed second-order reducibility functions ($k=3$) of the form

$$(\tilde{Z}_2)_{\alpha\beta\gamma\delta\varepsilon}=-3g\varepsilon_{\alpha\beta\gamma\delta\varepsilon}W_6(\varphi)\,,\quad(\tilde{Z}_2^{\mu\nu\rho})_{\alpha\beta\gamma\delta}=-\frac{1}{6}D_{[\alpha}\delta^\mu_\beta\delta^\nu_\gamma\delta^\rho_{\delta]}\,, \tag{G.26}$$
$$(\tilde{Z}_2^{\mu\nu\rho})_{\alpha\beta\gamma\delta\varepsilon}=-g\left(\frac{dW_6}{d\varphi}(\varphi)K^{\mu\nu\rho}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}+\frac{1}{6}\frac{dW_2}{d\varphi}(\varphi)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\phi_{\delta\varepsilon]}\right), \tag{G.27}$$
$$(\tilde{Z}_2^{\mu\nu\rho\lambda})_{\alpha\beta\gamma\delta}=-\frac{1}{12}gW_1(\varphi)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_{\delta]}\,,\quad(\tilde{Z}_2^{\mu\nu\rho\lambda})_{\alpha\beta\gamma\delta\varepsilon}=(Z_2^{\mu\nu\rho\lambda})_{\alpha\beta\gamma\delta\varepsilon}\,, \tag{G.28}$$
$$(\tilde{Z}_2^{\mu\nu\rho\lambda\sigma})_{\alpha\beta\gamma\delta\varepsilon}=-\frac{1}{40}gW_2(\varphi)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\,, \tag{G.29}$$

such that the corresponding on-shell, second-order reducibility relations are expressed by

$$(\tilde{Z}_1^\mu)(\tilde{Z}_2)_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}_1^\mu)_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon}+$$
$$+(\tilde{Z}_1^\mu)_{\alpha'\beta'\gamma'\delta'\varepsilon'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'\varepsilon'})_{\alpha\beta\gamma\delta\varepsilon}=$$
$$=-3g\frac{dW_6}{d\varphi}\frac{\delta\tilde{S}}{\delta H_\mu}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\,, \tag{G.30}$$

$$(\tilde{Z}_1^{\mu\nu\rho})_{\alpha'\beta'\gamma'}(\tilde{Z}_2^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta}+(\tilde{Z}_1^{\mu\nu\rho})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta}=$$
$$=-\frac{1}{3}g\frac{dW_1}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\lambda}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_{\delta]}\,, \tag{G.31}$$
$$(\tilde{Z}_1^{\mu\nu\rho\lambda})(\tilde{Z}_2)_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}_1^{\mu\nu\rho\lambda})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon}+$$
$$+(\tilde{Z}_1^{\mu\nu\rho\lambda})_{\alpha'\beta'\gamma'\delta'\varepsilon'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'\varepsilon'})_{\alpha\beta\gamma\delta\varepsilon}=$$
$$=\frac{1}{8}g\frac{dW_2}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\sigma}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\,, \tag{G.32}$$
$$(\tilde{Z}_1^{\mu\nu})_{\alpha'\beta'\gamma'}(\tilde{Z}_2^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta}+(\tilde{Z}_1^{\mu\nu})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta}=$$
$$=\frac{1}{2}g\left(\frac{d^2W_1}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^\rho}A_\lambda+\frac{dW_1}{d\varphi}\frac{\delta\tilde{S}}{\delta B^{\rho\lambda}}\right)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_{\delta]}\,, \tag{G.33}$$
$$(\tilde{Z}_1^{\mu\nu})(\tilde{Z}_2)_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}_1^{\mu\nu})_{\alpha'\beta'\gamma'\delta'\varepsilon'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'\varepsilon'})_{\alpha\beta\gamma\delta\varepsilon}+$$
$$+(\tilde{Z}_1^{\mu\nu})_{\alpha'\beta'\gamma'}(\tilde{Z}_2^{\alpha'\beta'\gamma'})_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}_1^{\mu\nu})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_2^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon}=$$
$$=\frac{1}{4}g\left(\frac{d^2W_2}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^\rho}\phi_{\lambda\sigma}+\frac{dW_2}{d\varphi}\frac{\delta\tilde{S}}{\delta K^{\rho\lambda\sigma}}\right)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}-$$
$$-3g\left(\frac{dW_6}{d\varphi}\frac{\delta\tilde{S}}{\delta\phi_{\mu\nu}}-\frac{d^2W_6}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^\rho}K^{\mu\nu\rho}\right)\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\,. \tag{G.34}$$

In a similar manner we obtain the deformed third-order reducibility functions ($k=4$) like

$$(\tilde{Z}_3^{\mu\nu\rho\lambda})_{\alpha\beta\gamma\delta\varepsilon}=-\frac{1}{24}D_{[\alpha}\delta^\mu_\beta\delta^\nu_\gamma\delta^\rho_\delta\delta^\lambda_{\varepsilon]}\,,\quad(\tilde{Z}_3^{\mu\nu\rho\lambda\sigma})_{\alpha\beta\gamma\delta\varepsilon}=\frac{1}{60}gW_1(\varphi)\,\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\,, \tag{G.35}$$

together with the accompanying on-shell, third-order reducibility relations

$$(\tilde{Z}_2^{\mu\nu\rho\lambda})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_3^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}_2^{\mu\nu\rho\lambda})_{\alpha'\beta'\gamma'\delta'\varepsilon'}(\tilde{Z}_3^{\alpha'\beta'\gamma'\delta'\varepsilon'})_{\alpha\beta\gamma\delta\varepsilon}=$$
$$=-\frac{1}{12}g\frac{dW_1}{d\varphi}\frac{\delta\tilde{S}}{\delta H^\sigma}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\,, \tag{G.36}$$
$$(\tilde{Z}_2^{\mu\nu\rho})_{\alpha'\beta'\gamma'\delta'}(\tilde{Z}_3^{\alpha'\beta'\gamma'\delta'})_{\alpha\beta\gamma\delta\varepsilon}+(\tilde{Z}_2^{\mu\nu\rho})_{\alpha'\beta'\gamma'\delta'\varepsilon'}(\tilde{Z}_3^{\alpha'\beta'\gamma'\delta'\varepsilon'})_{\alpha\beta\gamma\delta\varepsilon}=$$
$$=\frac{1}{6}g\left(\frac{d^2W_1}{d\varphi^2}\frac{\delta\tilde{S}}{\delta H^\lambda}A_\sigma+\frac{dW_1}{d\varphi}\frac{\delta\tilde{S}}{\delta B^{\lambda\sigma}}\right)\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_\gamma\delta^\lambda_\delta\delta^\sigma_{\varepsilon]}\,. \tag{G.37}$$

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